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*Understanding a Small Open Economy*

*Business Conditions Index*

**Jeffrey Sheen, Stefan Trück and Ben Wang**

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# UNDERSTANDING A SMALL OPEN ECONOMY BUSINESS CONDITIONS INDEX<sup>☆</sup>

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## Abstract

We estimate an unobservable domestic business conditions index for Australia using a variety of observable macroeconomic and financial variables, relating it to an unobservable external index involving external variables relevant to Australia. Our small open economy, dynamic factor model uses stock and flow variables arriving at mixed frequencies. We find important links between the domestic and external indices, consistent with the small open economy assumption. A business confidence measure and the terms of trade play significant roles for the indices, making them leading indicators. Financial variables matter only in severe macroeconomic episodes.

*Keywords:* Business conditions, Kalman filter, State-space model, Australia, Dynamic factor model, Mixed frequency

JEL Classification: E2, E3

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## 1. Introduction

We construct a small open economy version of a dynamic factor model that uses observable domestic variables and observable external driving variables for this economy. We use data for Australia, which is an excellent example of a relatively rich, small open economy. Our observable data includes macroeconomic and financial variables, and we apply this information in a dynamic factor model to obtain an optimally filtered estimate of an unobserved domestic business conditions index, as well as an unobserved external index representing the key external drivers of the domestic economy. Our estimation is based on a daily model, even though data on our observable variables arrive at different frequencies. Our unobservable indices are constructed using the Kalman filter, which deals optimally with missing daily observations by ignoring them. Our work builds on the closed economy modelling by Aruoba et al. (2009), who develop a business conditions index for the United States.

Our major contribution is to extend this research to a small open economy, and to provide a detailed analysis of how the data on observable variables add useful information to the

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index.<sup>1</sup> Among our results for the Australian case, we find that vital roles are played by a measure of business confidence for understanding domestic business conditions, and by the terms of trade for Australia's external index. Given that the Kalman filter utilizes newly arrived information by applying appropriate weights on predictive errors, these two observable variables play their important roles because of their predictive content. Our indices are thus leading indicators of economic outcomes. We find a number of significant channels linking the two indices, which support the small open economy assumption. In general, financial variables were particularly important during intense macroeconomic episodes, but not otherwise. Our results show that the external index was critical for the downturn in the domestic index during the recent financial crisis. In 2013, the external index remained strong, and yet the domestic index was weakening, suggesting undue pessimism in the state of business confidence.

### *Literature review*

Dynamic factor models have become popular in macroeconomics because they provide a powerful econometric tool for a parsimonious understanding of the co-movement between large numbers of time series. The basic hypothesis of the dynamic factor model is that there exists a small number of unobserved and common stochastic factors that can significantly explain the observed co-movements among a large number of time series.<sup>2</sup> The global financial crisis that began in 2008 revealed a strong need to integrate complex information about financial markets with macroeconomic data. Dynamic factor models provide a vehicle to meet this need.

Over the last decade, the literature has pointed out the importance of measuring economic activity at high frequency or potentially in real time, see e.g. Altissimo et al. (2001), Evans (2005), Giannone et al. (2008), Aruoba et al. (2009), Angelini et al. (2011). To try to meet this need, extensive resources in the private and public sector are devoted to providing accurate and timely forecasts of the state of the real economy. These forecasts are vital for business, finance and policy managers who have to take decisions in real time. The problem they face is that there is so much data on relevant variables, arriving at different times and frequencies, with some variables being stocks and others flows. Forecasters need to make sense of all this information, but the complexity of the data makes this very difficult. To create an effective framework for delivering metrics of economic activity in real time (generating what are called now-casts rather than forecasts) with data on relevant stock and flow variables arriving at different frequencies, the varied set of information needs to be integrated using some type of efficient multivariate filter.

We apply a mixed-frequency approach with the Kalman filter to integrate macroeconomic and financial variables in a coherent dynamic factor model. The dynamic factor model

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<sup>1</sup>Apart from extending the Aruoba et al. (2009) model to a small open economy with a second connected external index, we also provide full details of our estimates.

<sup>2</sup>For example, Bernanke et al. (2005) consider factors of macro aggregates that may be relevant for monetary policy; Kose et al. (2012) consider interactions between industrial, emerging and developing country factors; and Leu and Sheen (2011) focus on Australia in a context of the global and regional factors affecting its economic activity.

has recently been extended for the use of mixed-frequency data by (Mariano and Murasawa, 2003) and Aruoba et al. (2009). Since this approach will lead to a large amount of missing data for the variables at lower frequencies, a state-space representation of the problem is appropriate. This representation also lends itself to an effective way of dealing with a model that includes both stock and flow variables. For the estimation of these models, a Kalman filter can be used to construct high-frequency unobserved indicators, which will utilise efficiently information from the lower-frequency variables despite many missing data points.

Section 2 introduces the econometric model, giving details on the data we use, the operation of the Kalman filter in estimation of the dynamic factor model in state-space form, and the particular mixed-frequency, stock-flow model that we estimate. Section 3 presents results and Section 4 concludes.

## 2. The econometric model

### 2.1. The data

We follow Aruoba et al. (2009) in using the Kalman filter to handle stock/flow variables whose data arrive at mixed frequencies, extending their analysis to the small open economy. We select nine observable variables on the following five bases: first, we want to have both domestic and external variables for Australia; second, we want financial variables as well as real economic variables; third, we want variables that arrive daily, monthly and quarterly; fourth, we want both stock and flow variables; and fifth, we want to include variables that have good predictive content.

We use five domestic data series sourced from the RBA. The domestic yield spread or premium between a 10 year Australian Treasury bond and a 3 month Treasury bill (financial, daily and a stock measure),  $ys_t$ , tends to be countercyclical because investors tend to avoid risk in bad times, but procyclical because the central bank acts to lower shorter rates in these times. Hours worked (real, monthly and a flow),  $hrs_t$  is a real procyclical variable reflecting total labour market employment. The NAB survey-based business confidence index (real, quarterly and a stock),  $bc_t$  is a forward-looking and thus predictive indicator of general business sentiment in Australia, which is procyclical. Real GDP (real, quarterly and a flow),  $gdp_t$  measures total output across all sectors in Australia. Job vacancies (real, quarterly and a stock),  $vac_t$ , reflect the demand for labour at the extensive margin, and is procyclical.

We use four external data series sourced from the RBA, ABS, Datastream and FRED. The real trade-weighted index (real, quarterly and a stock measure),  $twi_t$ , has positive income but negative substitution trade effects as does the terms of trade (real, quarterly and a stock),  $tot_t$ , which may have good predictive content for external drivers of the Australian economy. An export-weighted world real GDP index (real, quarterly and a flow)<sup>3</sup>,  $wgdp_t$ , measures the positive effect of world economic activity on Australia through the external index. The TED spread between the 3 month US Treasury bill rate

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<sup>3</sup>We construct the export weights from ABS Cat.5360. The top 10 exporters across every quarter account for between a minimum 63% and a maximum 82% of total Australian exports. 27 countries appeared in the top 10 exporters across the sample period. Out of these 27 countries, we count the

and its LIBOR rate (financial, daily and a stock),  $ted_t$ , proxies an external risk premium from world credit markets, which is an external driver that should negatively impact on Australia.

In line with Aruoba et al. (2009), we detrended our non-stationary series using a third-order polynomial trend using the particular variable's frequency of observation. This form of detrending is sufficiently flexible to deal with most non-stationary macroeconomic series. The series that we detrend in this way are  $hrs_t$ ,  $gdp_t$ ,  $vac_t$ ,  $twi_t$ ,  $tot_t$  and  $wgdp_t$ , and in Appendix B, we show the original data series with the estimated time trends.<sup>4</sup> We then standardize all of our variables to have a zero mean and unit variance, so that we can easily compare the effects of our observable variables in the estimated model. Appendix A shows the plot of the transformed data series used in estimation.

Our sample ranges from 01/01/1986 to 02/05/2013. However for two observed variables, the available data begins later: the business confidence index begins in 30/09/1989, and our weighted world GDP variable for Australia begins on 15/02/1988. All of our data is measured as of the end of period, except for world GDP, which is mid-period.

## 2.2. The Kalman filter in a general dynamic factor model

The general form of a state-space dynamic factor model at the highest frequency amongst observed variables is:

$$X_t = AC_t + RX_{t-1} + We_t \quad e_t \sim N(0, P) \quad (2.1)$$

$$Y_t = GD_t + BX_t + \eta_t \quad \eta_t \sim N(0, \sigma^2) \quad (2.2)$$

where  $X_t$  represents the unobserved states or factors,  $C_t$  the exogenous variables that drive the states,  $Y_t$  the observed data series, and  $D_t$  the exogenous variables (which will be the lagged states in our case) that drive the observed data.  $e_t$  and  $\eta_t$  are the innovation and measurement errors respectively, which are assumed to follow normal distributions with a 0 mean and variance  $P$  and  $\sigma^2$ . The parameter matrix,  $B$ , represents the loadings of the factors,  $X_t$ , in relation to the relevant observed variables. Equations (2.1) and (2.2) are normally referred to as the state and measurement equations. In such a dynamic factor model, an identification assumption needs to be made in regard to the variance of any factor,  $X_t$ . We restrict the variance of each factor to be the average of the estimated variances of its component observable variables.

Denote  $X_{t|t-1}$  and  $\Sigma_{t|t-1}$  as the predicted states and their associated variance at time  $t$  given time  $t-1$  information,  $X_{t|t}$  and  $\Sigma_{t|t}$  as the updated values given time  $t$  information,

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number of occurrences when each appears in the top 10 exporters in any quarter and select the highest 10 among the 27. (The cut-off country appeared 61% of the time and the 11<sup>th</sup> country appeared 42%). The top 10 countries were Japan, Korea, NZ, US, Taiwan, China, Singapore, UK, Indonesia and Hong Kong. The quarterly real GDP data for these countries were obtained from Datastream, and for each quarter, the associated export weights were extracted for these countries. The aggregate Australian export-weighted world real GDP index was constructed by multiplying the weights with the deseasonalized real GDP (detrended with a third order polynomial) of the respective countries, and then adding up.

<sup>4</sup>As a robustness check, we estimated the model with growth rates instead of time-detrended series, but this did not alter our conclusions.

the Kalman filter algorithm is given by the following six equations:

$$X_{t|t-1} = AC_{t|t-1} + RX_{t-1|t-1} \quad (2.3)$$

$$\Sigma_{t|t-1} = R\Sigma_{t-1|t-1}R' + WPW' \quad (2.4)$$

$$X_{t|t} = X_{t|t-1} + K_tv_t \quad (2.5)$$

$$\Sigma_{t|t} = \Sigma_{t|t-1} - K_tB\Sigma'_{t|t-1} \quad (2.6)$$

$$K_t = \Sigma_{t|t-1}B'(\sigma^2 + B\Sigma_{t|t-1}B')^{-1} \quad (2.7)$$

$$v_t = (Y_t - GD_t - BX_{t|t-1}) \quad (2.8)$$

where  $K_t$  is the ‘Kalman gain’ and  $v_t$  is the prediction error when using the previous period value of the state. The first two equations make predictions according to (2.1) and the next two update the predictions conditional on newly arrived data in  $Y_t$ . Since the Kalman filter algorithm is an iterative process whereby the unobserved states are updated whenever new information becomes available, it can be interpreted as a Bayesian learning process (see Meinhold and Singpurwalla (1983)). Combining (2.3) with (2.5) and noting that  $X_t = X_{t|t}$  gives:

$$X_t = AC_t + RX_{t-1} + K_tv_t \quad (2.9)$$

Equation (2.9) allows a linear regression or least squares learning interpretation of the Kalman filter. The estimation of the unobserved states  $X_t$  has a stochastic intercept  $AC_t + RX_{t-1}$  and a slope  $K_t$  for the prediction error  $v_t$ .<sup>5</sup> The Kalman gain will tend to be larger for variables with a lower predictive error variance, and thus those that have good predictive content will be favoured by the filter.

The particular model that we present next has no exogenous variables,  $C_t$ , in the state equation, and only uses lagged observables,  $Y_{t-1}$  for  $D_t$  in the measurement equation. Replacing the resulting (2.1) in (2.2) yields a restricted reduced form VARMA(2,1) model for estimation.

### 2.3. The model

For our model, we introduce two state variables: an unobserved domestic business conditions index for Australia ( $X_t^d$ ); and an unobserved external conditions index ( $X_t^f$ ) as it applies to Australia. This external index does not represent the world economy in general. Instead it represents the the key external drivers of the domestic economy.

Our model utilizes variables that arrive at daily, monthly and quarterly intervals. The dynamic model that we estimate is thus set up at the highest frequency—daily. Since our data consists of both stock and flow variables, with the latter arriving at both monthly (domestic only) and quarterly (domestic and external) intervals, we introduce three further unobserved states ( $C_t^{dM}$ ,  $C_t^{dQ}$  and  $C_t^{fQ}$ ) to represent the cumulating component of the domestic and world indices. As Aruoba et al. (2009) explain in an online appendix,

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<sup>5</sup>Assuming  $X_t$  and  $v_t$  are jointly normal,  $\text{Cov}(X_t, v_t) = \Sigma_{t|t-1}B'$  and  $\text{Var}(v_t) = \sigma^2 + B\Sigma_{t|t-1}B'$ . Therefore the Kalman gain matrix  $K_t$  is equivalent to  $\text{Cov}(X_t, v_t)\text{Var}(v_t)^{-1}$ , which is the least squares coefficient.

this is an efficient construct for handling mixed stock and flow data that arrive at different frequencies.

We assume an AR(1) process for the domestic business conditions indices,  $X_t^d$  and  $X_t^f$ , and the observed data series. We assume the lagged external index can impact on the current domestic index, but not vice versa because the Australian economy is assumed small. The five observed domestic variables are assumed to be associated with the domestic index,  $X_t^d$ , while the four observed external variables are associated with the external index,  $X_t^f$ . In addition, we allow two of the external variables,  $twi_t$  and  $tot_t$  to be associated also with the domestic index. This is because Australia's real trade-weighted exchange rate index and the terms of trade may be determined in some part by the domestic economy.

With these assumptions, the exogenous component,  $C_t$ , in the state equation (2.1) becomes a null matrix and  $D_t$  in the measurement equation (2.2) contains the previous day's data for the observable variables. Writing our daily model as in equation (2.1) and (2.2) expressed in matrix form gives:

$$\begin{aligned}
\begin{bmatrix} X_t^d \\ C_t^{dM} \\ C_t^{dQ} \\ X_t^f \\ C_t^{fQ} \end{bmatrix} &= \begin{bmatrix} \rho_d & 0 & 0 & \phi & 0 \\ \rho_d & \theta_d^M & 0 & 0 & 0 \\ \rho_d & 0 & \theta_d^Q & 0 & 0 \\ 0 & 0 & 0 & \rho_f & 0 \\ 0 & 0 & 0 & \rho_f & \theta_f^Q \end{bmatrix} \begin{bmatrix} X_{t-1}^d \\ C_{t-1}^{dM} \\ C_{t-1}^{dQ} \\ X_{t-1}^f \\ C_{t-1}^{fQ} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^f \end{bmatrix} \quad (2.10) \\
\begin{bmatrix} yX_t \\ hrs_t \\ bc_t \\ vac_t \\ gdp_t \\ twi_t \\ tot_t \\ wgdpt \\ ted_t \end{bmatrix} &= \begin{bmatrix} \gamma_{ys} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_{hrs} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{bc} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{vac} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_{gdp} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_{twi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{tot} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{wgdpt} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{ted} \end{bmatrix} \begin{bmatrix} yX_{t-1} \\ hrs_{t-1} \\ bc_{t-1} \\ vac_{t-1} \\ gdp_{t-1} \\ twi_{t-1} \\ tot_{t-1} \\ wgdpt_{t-1} \\ ted_{t-1} \end{bmatrix} \\
&+ \begin{bmatrix} \beta_{ys} & 0 & 0 & 0 & 0 \\ 0 & \beta_{hrs} & 0 & 0 & 0 \\ \beta_{bc} & 0 & 0 & 0 & 0 \\ \beta_{vac} & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_{gdp} & 0 & 0 \\ \beta_{twi}^d & 0 & 0 & \beta_{twi}^f & 0 \\ \beta_{tot}^d & 0 & 0 & \beta_{tot}^f & 0 \\ 0 & 0 & 0 & 0 & \beta_{wgdpt} \\ 0 & 0 & 0 & \beta_{ted} & 0 \end{bmatrix} \begin{bmatrix} X_t^d \\ C_t^{dM} \\ C_t^{dQ} \\ X_t^f \\ C_t^{fQ} \end{bmatrix} + \begin{bmatrix} \eta_t^{ys} \\ \eta_t^{hrs} \\ \eta_t^{bc} \\ \eta_t^{vac} \\ \eta_t^{gdp} \\ \eta_t^{twi} \\ \eta_t^{tot} \\ \eta_t^{wgdpt} \\ \eta_t^{ted} \end{bmatrix} \quad (2.11)
\end{aligned}$$

where  $\rho_d$  is the persistence parameter of the unobserved domestic business conditions index  $X_t^d$ ;  $\rho_f$  measures the persistence of the external conditions index;  $C_t^M$  and  $C_t^Q$  are the monthly and quarterly accumulated business conditions indices with  $\theta^M$  and  $\theta^Q$  as the respective binary time indicators (which will be discussed in more detail in section 2.3.2);  $\epsilon_t^d$  and  $\epsilon_t^f$  are the state innovations for the domestic and external index respectively; the  $\gamma_i$  measure the persistence of the observable variables (for  $i \in \{ys, hrs, bc, vac, gdp, twi, tot, wgdpt, ted\}$ ); the  $\eta_t^i$  are measurement or idiosyncratic errors



with variance  $\sigma_t^i$ ; and the  $\beta_i$  measure the respective contributions of the unobserved states to the observed variables.<sup>6</sup>

Two challenges arise in implementing the Kalman filter procedure because of the nature of the data we use: not every series has data available in every daily period, and some of our variables are measured flows while others are stocks. We discuss these challenges next in turn.

### 2.3.1. Handling mixed frequency data

Since our data are observed at various frequencies, the dimension of the number of actually observed data series,  $Y_t$  and  $Y_{t-1}$ , across days constantly changes size. Our objective is to use all available information to make daily inferences on the unobserved states,  $X_t$ . Therefore, we adjust the dimension of (2.10) and (2.11) in each period according to the number of observable variable with available data.

### 2.3.2. Handling stock/flow data

Stock data are snapshots of the measured variable at a given point in time, whereas flow data represent an accumulation over a given period. For example, the yield spread, business confidence and the vacancy rate are stock data, but hours worked and real GDP are flow data (monthly,  $M$ , and quarterly,  $Q$ , respectively). Since our domestic business and external conditions indices are daily indices, we need to accumulate them over a month for hours worked and over a quarter for real GDP and weighted world GDP.

For  $i \in (M, Q)$  and  $j \in (d, f)$ , the time aggregation follows:

$$\begin{aligned} C_t^{ji} &= \theta^i C_{t-1}^{ji} + X_t \\ &= \theta^i C_{t-1}^{ji} + \rho X_{t-1} + e_t^j \end{aligned} \quad \text{where: } \theta^i = \begin{cases} 0 & \text{if } t \text{ is the day when new data arrives} \\ 1 & \text{otherwise} \end{cases} \quad (2.12)$$

which is as seen in equation (2.10). Here  $X_t$  represents the indices for domestic business and external conditions. Thus the  $C_t^{ji}$  indices accumulate on a daily basis according to the persistence of the relevant  $X_t$ , and their own persistence when their own observation is recorded. The accumulated monthly and quarterly aggregate indices  $C^{dM}$ ,  $C^{dQ}$  and  $C^{fQ}$  then drive the observed flow variables, as in equation (2.11).

## 2.4. Estimation

As a by-product of the Kalman filter, the time  $t$  log-likelihood ( $L_t$ ) of the model can be evaluated from the prediction error  $v_t$ . Denoting the variance of the prediction error as  $\Psi = \sigma^2 + B\Sigma_{t|t-1}B'$ , we have:

$$\log L_t = -\frac{1}{2} \left( N \log 2\pi + \log |\Psi| + v_t \Psi^{-1} v_t' \right) \quad (2.13)$$

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<sup>6</sup>As shown by equation (C.5) in Appendix C, the Kalman filter uses  $\beta_i$  as a relative weight along with the variance of other variables to determine how the unobserved states are updated with newly arrived information for observable variable  $i$ . The sign of  $\beta_i$  dictates whether the variable contributes positively or negatively to the states. Therefore  $\beta_i$  measures how the domestic index contributes to the observables (if we know the index and are looking at equation (2.11)), or as a sign indicator of the correlation between the variable  $i$  and the unobserved index.

where  $N$  is the number of observations at time  $t$ . Thus maximizing the likelihood is equivalent to minimizing the prediction errors,  $v_t$ . We first use a simplex method to fine tune the starting values for 20 iterations, then switch to a quasi-Newton method with BFGS updates on the Hessian matrix for the rest of the estimation. Unlike Aruoba et al. (2009), we restrict the variance of the state innovations,  $P$ , to be in the same domain as the variance of the measurement errors. In particular, the variance of  $\epsilon_t^d$  is set at the mean of the measurement error variances that are directly relevant to the domestic business conditions index, and the variance of  $\epsilon_t^f$  is restricted to be the mean of the measurement error variances directly relevant to the external index.

### 3. Results

Table 1 shows the estimated parameter values associated with the optimally estimated unobserved states.<sup>7</sup> Both the domestic  $\rho_d$  and external index  $\rho_f$  are highly significant and represent high daily persistence. This high persistence is not surprising in a daily model. For example, the estimate for  $\rho_d$  is 0.9968 on a daily basis. Over a 90-day quarter, this would work out to be  $0.9968^{90}$  or 0.75.

There are three channels in this model through which external economic conditions have an impact on the domestic economy:  $\phi$ , which measures the direct link between the lagged external and the domestic economy index, and its point estimate is small but statistically significant; and  $\beta_{twi}^d$  and  $\beta_{tot}^d$ , which measure the association of the domestic indicator (in addition to the external indicator) with the trade-weighted index and the terms of trade. Our estimates indicate that a rise in both the trade-weighted index and the terms of trade are significantly associated with an improvement in the domestic business condition index, possibly representing an income and wealth effect. Nevertheless the total effect of  $twi$  and  $tot$  on the domestic index also depends on the negative, possibly substitution, effects they have on the external index, which subsequently raises the domestic index after a lag (through  $\phi$ ). The net effect will be shown to be negative in the impulse response analysis.

All observed variables have significant and high daily persistence parameters ( $\gamma$ s) except for the business confidence variable. Since the daily persistence of economic activity variables ( $gdp$ ,  $hrs$  and  $vac$ ) are close to unity and  $bc$ 's persistence is insignificantly different from zero, the strong predictive power of  $bc$  for  $X_t^d$ , evident from its relatively high loading, indicates that the business confidence variable is a good predictor for the growth in the economic activity variables.

The loading parameters ( $\beta$ s) of the relevant observables for the domestic index generally have the expected signs. For example, an increase in the domestic index is significantly associated with a rise in business confidence by 0.2139, which is at least four times bigger in absolute value than all other relevant loadings. Only the loading for hours worked is insignificant (above 10%). The yield spread has a weakly significant (at 10%) negative

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<sup>7</sup>Evidently, from the likelihood function (2.13) and the prediction errors (2.8), the parameter values of the loadings can only be jointly identified with the estimated unobserved states, or factors. In particular, if we reverse the sign of both the parameter matrix  $B$  and the states  $X_t$ , we do not change the forecast error  $v_t$ , thus arriving at the identical likelihood.

loading on to the domestic index, indicating that the negative risk premium effect weakly dominates the positive effect that would come from monetary policy. The small though significant loading for  $gdp$  indicates that  $gdp$  has minimal predictive content for other observed variables, which is not surprising since  $gdp$  is explicitly a measure of past activity.

The loadings for the external index have the correct expected signs and all are significant (below 10%). For example, an increase in Australia’s external index is associated with a reduction in the terms of trade by a loading of -0.1076 (significantly at 1%). This loading is at least four times greater in absolute value than for any other relevant variable.

All but two of the idiosyncratic standard deviations are significantly greater than 0. The two that are not are business confidence and the terms of trade. This indicates a close relationship between business confidence and the domestic index, and between the terms of trade and the the domestic and external indices. As shown in Appendix C, the estimated standard deviations of the measurement errors, and in turn the predictive errors, play a significant role in determining how the Kalman filter optimally updates the unobserved states. The intuition is that if one observed variable has a near zero standard deviation, the series does not have an idiosyncratic stochastic component and its stochasticity is mimicked closely by the unobserved states. Since both business confidence and terms of trade are stochastic and have insignificant measurement error standard deviations (at greater than 10%), our indices are updated heavily by these two variables. Thus they have significant predictive content, giving substance to the notion that the estimated indices have a leading nature.

Figure 1 shows the optimally filtered and smoothed domestic business conditions index  $X_t^d$  (blue thick line) with its 95% confidence interval (red thin lines) for the Australian economy.<sup>8</sup> It shows that Australia experienced two major economic downturns, one between 1989 and 1991 and the other one being the GFC from late 2007 to 2008. There were several mild slowdowns between 1997 and 1998 and between late 1999 to late 2000, presumably due to the effects of the Asian financial crisis and that associated with the dotcom crash in stockmarkets. The index shows the Australian economy beginning to slip into a small downturn after 2009, but recovering by May 2013. The narrowing of the confidence band after 1989 occurred simply because the observed business confidence indicator only became available after that date, and this variable then played a large and significant role in regard to the domestic business conditions index.

Figure 2 brings together the two filtered indices. The key conclusion from this figure is that there is a significant correlation between the lagged external index and the cur-

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<sup>8</sup>The filtered indices are appropriate for conditional forecasting, while the smoothed ones are appropriate for within-sample estimation. The filter estimates the states at time  $t$  by using all available information up to time  $t$ . The smoother estimates the states using all available information across the whole sample. The classical fix interval smoother (for example as described by Hamilton (1994)) is not suitable for our model because we have flow as well as stock variables. This algorithm requires inversions of the variance-covariance matrix of the predicted state density  $\Sigma_{t|t-1}$ , (defined in (2.4)), which is necessarily singular whenever new data on flow variables become available. We use the alternative smoother algorithm described by Durbin and Koopman (2002), which does not require inversion of  $\Sigma_{t|t-1}$ . As shown on page 71 of Durbin and Koopman (2002), the two algorithms are equivalent if there are no inversion problems.

Parameter	Mean	Standard Error	Significance
$\rho_d$	0.9968	0.0011	***
$\rho_f$	0.9950	0.0020	***
$\phi$	0.0038	0.0014	***
$\gamma_{ys}$	0.9984	0.0007	***
$\gamma_{hrs}$	0.9541	0.0216	***
$\gamma_{bc}$	0.1523	0.1026	
$\gamma_{vac}$	0.9838	0.0280	***
$\gamma_{gdp}$	0.9766	0.0299	***
$\gamma_{twi}$	0.8478	0.0513	***
$\gamma_{tot}$	0.7963	0.0808	***
$\gamma_{wgdp}$	0.7538	0.0580	***
$\gamma_{ted}$	0.9861	0.0021	***
$\beta_{ys}$	-0.0004	0.0002	*
$\beta_{hrs}$	0.0003	0.0002	
$\beta_{bc}$	0.2139	0.0222	***
$\beta_{vac}$	0.0502	0.0100	***
$\beta_{gdp}$	0.0003	0.0001	***
$\beta_{twi}$	-0.0275	0.0140	**
$\beta_{tot}$	-0.1076	0.0199	***
$\beta_{wgdp}$	0.0004	0.0002	*
$\beta_{ted}$	-0.0017	0.0007	***
$\beta_{twi}^d$	0.0483	0.0157	***
$\beta_{tot}^d$	0.0305	0.0155	**
$\sigma_{ys}$	0.0548	0.0005	***
$\sigma_{hrs}$	0.3451	0.0135	***
$\sigma_{bc}$	0.0247	0.1740	
$\sigma_{vac}$	0.2514	0.0184	***
$\sigma_{gdp}$	0.2997	0.0206	***
$\sigma_{twi}$	0.4679	0.0319	***
$\sigma_{tot}$	0.0732	0.1141	
$\sigma_{wgdp}$	0.5692	0.0405	***
$\sigma_{ted}$	0.1519	0.0013	***

Table 1: Estimated parameters

rent domestic index. However the connection seems much weaker in the years after the financial crisis in 2008. The two indices coincide (with a lag) through 2008-9, which implies that the immediate crisis that Australia experienced was almost entirely externally driven. From 2011 to 2013, the domestic index was persistently negative despite an improving and substantially positive external index, which suggests undue pessimism in domestic business confidence.

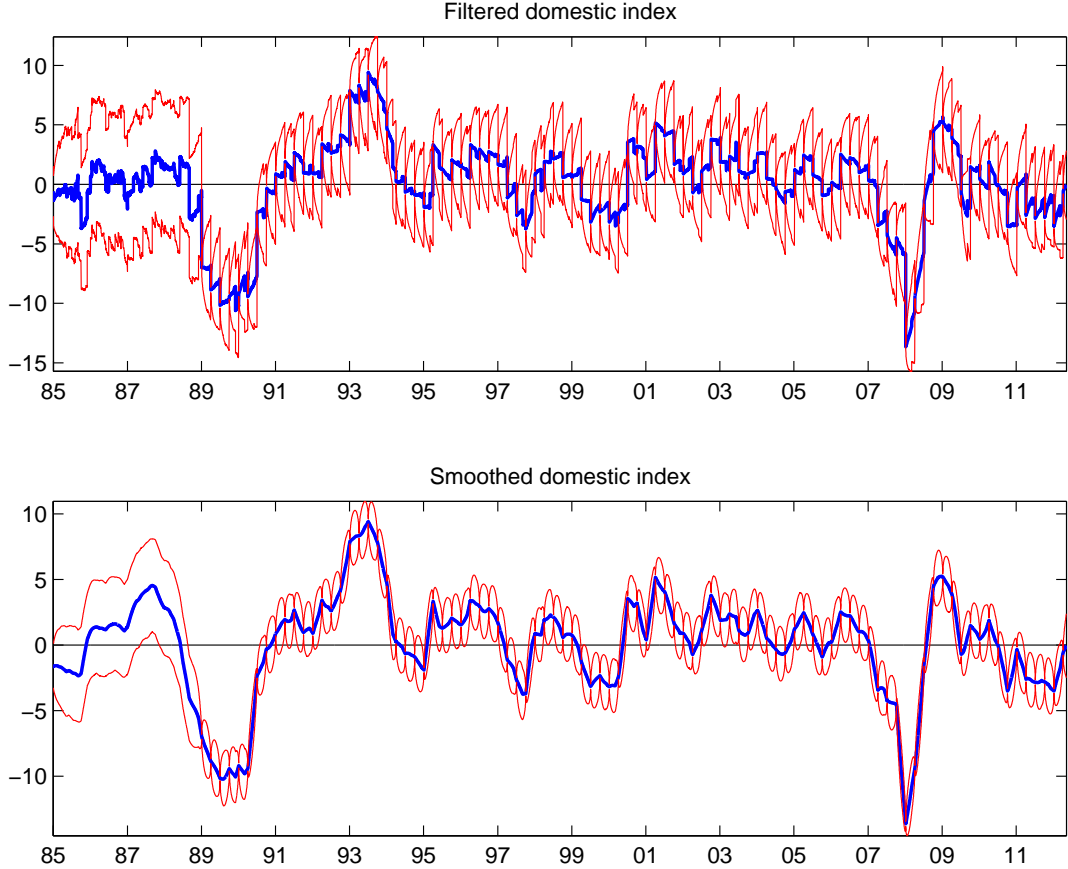


Figure 1: Filtered and smoothed domestic business conditions index and 95% confidence interval

### 3.1. Decomposing the relative contributions

The unobserved states are updated with observed information according to the optimally weighted prediction errors. Evident from the correction equation (2.5), the Kalman gain matrix,  $K$ , contains information on the optimal time-varying weighting of the prediction errors to the domestic business and external conditions indices,  $X_t$ . In particular, the dot product of the first and fourth row of  $K_t$  and the prediction error vector  $v_t$  measures the respective contribution of each prediction error to  $X_t^d$  and  $X_t^f$ . Therefore we can decompose the indices into the contributions from each of the observed series. Dropping the exogenous component which does not appear in our model, and iterating (2.9) backwards and calculating the dot products and left-multiplying by matrix  $R^s$  gives:

$$S_t^* = \sum_{s=0}^t R^s (K_{t-s} \cdot v'_{t-s}) \quad (3.1)$$

where  $v_{t-s}$  is a  $9 \times 1$  vector,  $K_{t-s}$  is a  $5 \times 9$  matrix and  $R^s$  is a  $5 \times 5$  matrix.  $X_t^*$  becomes a  $5 \times 9$  matrix, with each row comprising the decomposition of the aggregate contribution from the particular observable's data series at time  $t$ . This gives a structural interpretation of the otherwise mechanical index, conditional on the model and data we use. From equation (3.1), the extent of the contributions depends on both the relative weights and the prediction errors. The intuition for the latter is that if the dynamic factor model has a serious mistake in prediction for an observation, the model should

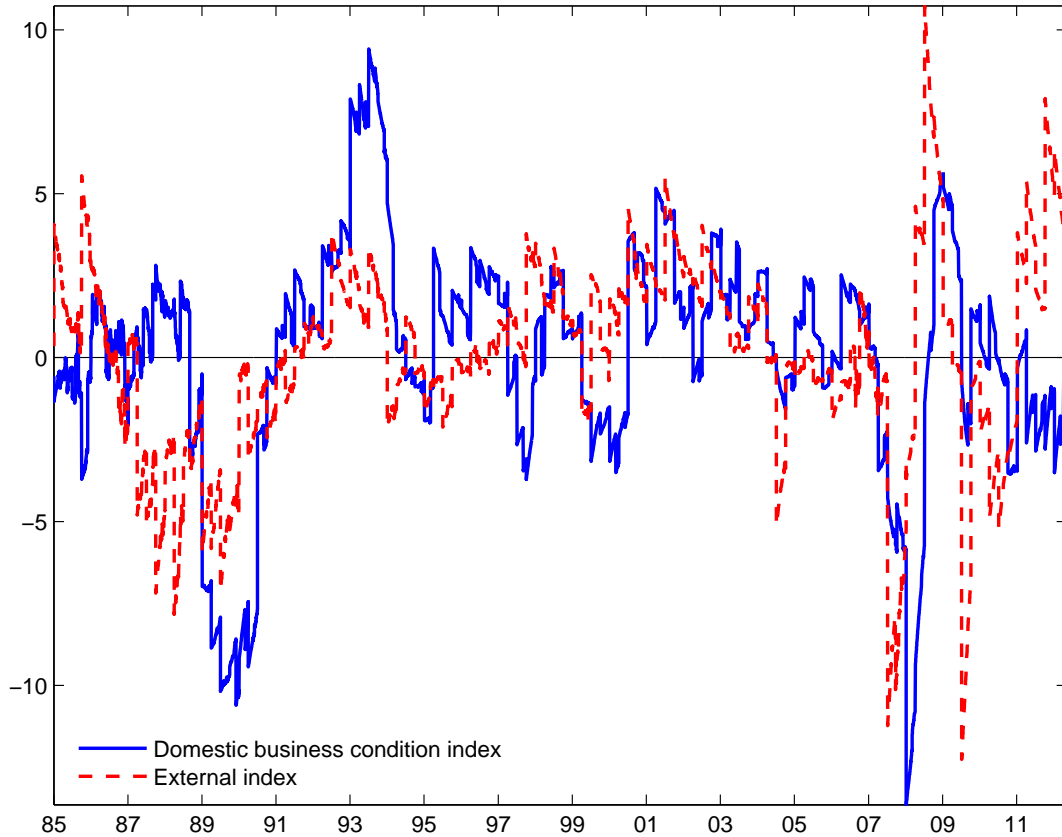


Figure 2: Optimally filtered domestic business and external conditions indices

learn from the mistake, and that observation will thus contribute more to the updated states. If that variable has a high  $\beta$  or a low measurement error variance relative to other observable variables, its relative Kalman weight in  $K$  will be large, which will amplify its contribution to the updated states.

### 3.1.1. Contributions to the domestic business conditions index

Figure 3 shows the time series of individual contributions to the domestic business conditions index,  $X_t^d$ . The first sub-plot shows the filtered and then smoothed domestic business conditions index. At each point in time, the sum of the contributions of each observed variable shown in the other subplots yields  $X_t^d$ .<sup>9</sup> This Figure shows that business confidence,  $bc$ , after it became available in September 1989, played a dominant role in explaining the estimated movements in the domestic business conditions index. The 1990-91 recession was mainly associated with a decline in business confidence, job vacancies and a rising terms of trade (that worsened domestic business conditions). In the heat of the global financial crisis in 2008, the large real depreciation of the A\$ and the jump

<sup>9</sup>Note that these contributions measure the effects of the prediction errors on the index, and thus include the sign and size of the relevant  $\beta$ . For example, the negative contribution of the TED spread in the 2008 crisis occurred because the TED spread rose significantly. Also, the apparently negligible contributions of  $hrs$  and  $gdp$  is not what they appear because these are flow variables, and so their daily contributions should be aggregated over the relevant month and quarter. Both of these flow variables played a much bigger role before the business confidence variable became available in September 1989.

in the TED spread also play a large role. In the next 5 years, the surge in the terms of trade, and to a lesser extent the gradual recovery of Australia's trading partners' GDP, play a role in describing the movements in the Australian domestic business conditions index.

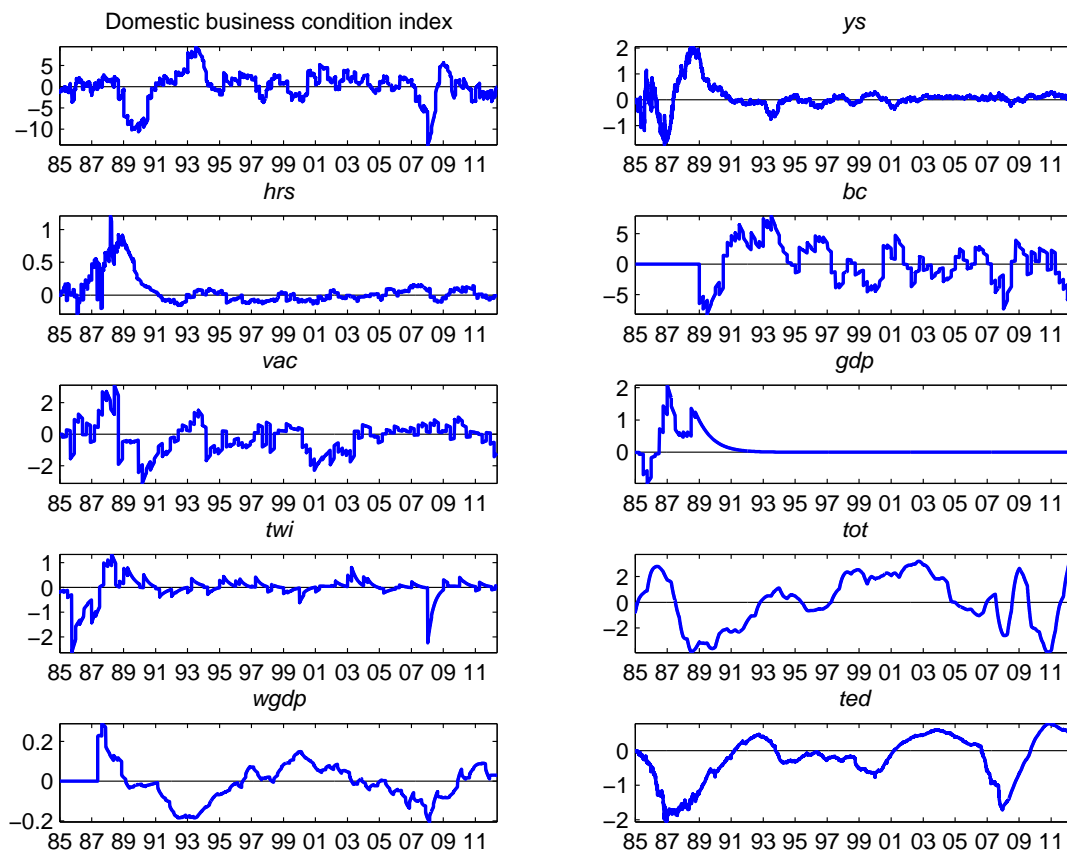


Figure 3: Accumulated period contributions of the prediction errors to the domestic index

The upper panel of Figure 4 separates the aggregated contributions of internal and external observed variables to the domestic business conditions index,  $X_t^d$ . The internal contributions to the index (blue solid line) give the sum of the contributions from the yield slope, hours worked, business confidence, job vacancies and real GDP, while the external contributions to the index (red dashed line) involves the terms of trade, real TWI, export-weighted world real GDP and the TED spread. As explained earlier, external variables have three channels for driving the domestic business conditions index: through  $\phi$ ,  $\beta_{twi}^d$  and  $\beta_{tot}^d$ , all of which are statistically significant.

There are three interesting features from this decomposition.

First, the external contributions to the domestic business conditions index led the internal contributions during the late 1980s early 1990s recession. The decline due to the external variables was primarily due to the negative impact of the terms of trade in 1988, which reduced the demand for Australian exports, as well as an increase in the TED spread in 1987, which led to tighter global credit markets. Subsequently, and to a greater effect,

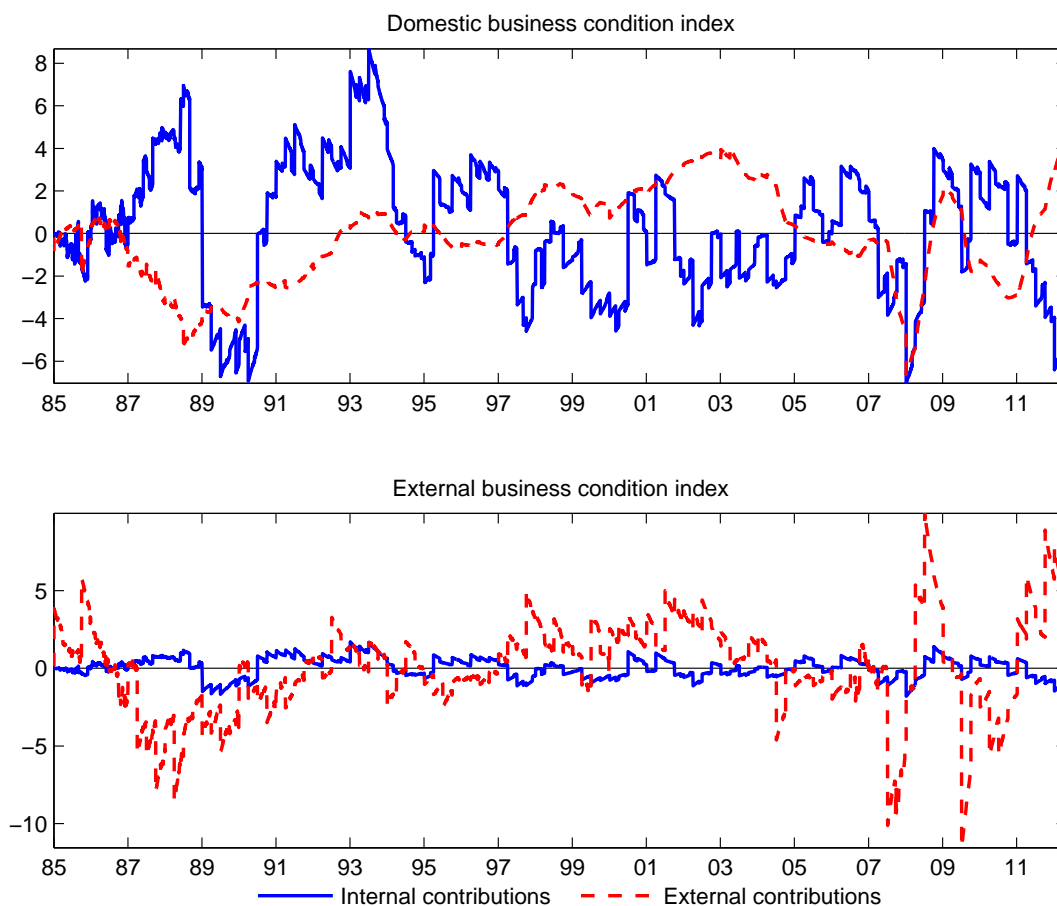


Figure 4: Aggregate contributions of observable internal and external variables to the unobservable domestic business and external indices

monetary policy was substantially tightened to bring down inflation, and as a result the yield spread turned negative.<sup>10</sup> These events sparked a crisis in Australian business confidence, leading to a freezing on hiring and a reduction in hours worked.

Second, during the global financial crisis that began in 2008, the deterioration of the external drivers was accompanied contemporaneously by a reduction in the internal drivers. The large increase in the TED spread led to a domestic credit market crunch and a decline in business confidence, and thus domestic business conditions. In the heat of the crisis in late 2008, international capital markets made a dash for traditional safe havens (mainly US dollars and gold), and so Australia suffered a large depreciation of the TWI and a fall in the terms of trade. This improved the competitiveness of Australia's exports, which was further supported by persistent demand from China for Australia's exports despite the global slowdown. These factors quickly restored domestic business conditions.

Third, after 2011, the internal contributing factors deteriorated again mainly through the combination of falling business confidence and thus fewer vacancies. In contrast, until

<sup>10</sup>In fact, this was the only period in our sample in which monetary policy operating through  $ys$  made a substantial contribution to the domestic business conditions index.



early-2013, the external contributing factors improved significantly, partially alleviating the negative impact of business pessimism within Australia.

To further understand the disaggregated contributions, the upper panel of Figure 5 shows the interpolated<sup>11</sup> Kalman gains or weights ( $K_t$  in (2.7)) for each observable series at each point in time for the domestic business conditions index. These gains when multiplied by the prediction errors ( $v_t$  from (2.8)) lead to an updating of the state variables as seen in (2.5). As explained above, the Kalman gain represents the time-varying ‘learning’ parameter for that period’s prediction errors, and its relative size is large if the  $\beta$  of the variable is large and/or if the variances of the all the other relevant observable variables are relatively large (while the sign is determined by the respective  $\beta$ )<sup>12</sup>. The time-varying nature of the Kalman gain stems mainly from two features in our model: the time aggregation of the flow variables and the mixed frequency of observations that alters the optimal weighting structure on observables when updating the states. The Kalman gain weights of all observed variables jump in September 1989 because that is the first date of available information on business confidence—the resulting patterns reveal the Kalman filter optimally downgrading the importance of other observables after the business confidence variable became available.<sup>13</sup> The seemingly seasonal pattern was simply due to the time aggregation of both monthly and quarterly flow variables.

Figure 6 shows the interpolated prediction errors  $v_t$  for each observable series at each point in time. These prediction errors can be interpreted as the performance of in-sample forecasts, upon which the maximum likelihood estimator and the Kalman filter aim to optimize. In most cases, the states ‘learn’ relatively more from business confidence because it has relatively good predictive value given its low predictive variance. It is interesting that the yield slope was harder to predict only early in the sample, and especially not predictable during the crisis in 2008. In general, it is hard to forecast external variables well, particularly during crisis periods.

### 3.1.2. Contributions to the external conditions index

Figure 7 shows the time series of individual contributions to the external conditions index,  $X_t^f$ . The first sub-plot presents the filtered and then smoothed external index. At each point in time, the sum of the contributions of each observed variable shown in the other subplots yields  $X_t^f$ . It is not surprising to see that the external index ‘learns’ most of its movement from the terms of trade, suggesting that relative price movements of exports to imports cannot be well-predicted given the information extracted from the data series we consider, and that it is probably heavily driven exogenously in overseas markets. The TED spread contributes to the external index significantly only during the crisis periods. The sharp depreciation of the trade weighted index probably due to a ‘flight to quality’ during the 2008 crisis was not predicted by our model.

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<sup>11</sup>Interpolation means that when no new information is available, the prediction error remains what it was at the most recent observation.

<sup>12</sup>See Appendix C for an explanation of this statement.

<sup>13</sup>When the business confidence measure was excluded across the whole sample, we found that vacancies assumed the major role for the domestic index. This is likely because vacancies represent a forward-looking measure of upcoming employment and output.

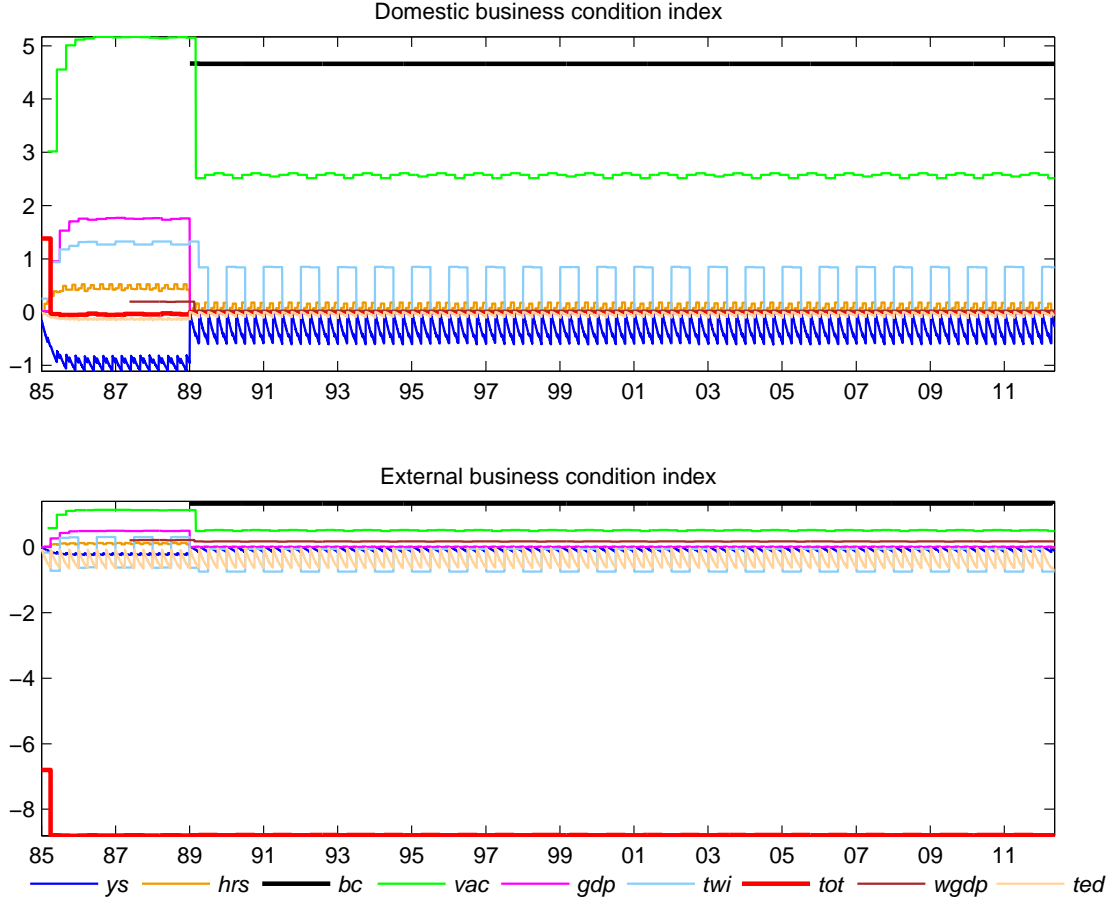


Figure 5: Interpolated Kalman gain weights for the domestic business and external index

The lower panel of Figure 4 separates the internal and external contributions to the external conditions index,  $X_t^f$ . It demonstrates the small open economy nature of the Australian economy, since internal variables do not contribute much to the external conditions index.<sup>14</sup>

The lower panel of Figure 5 shows the Kalman weights for each series at each point in time for the external index. The terms of trade make the biggest contribution by a large margin. As expected, the introduction of business confidence in 1989 barely alters the allocation of optimal weights for the foreign variables.

### 3.2. Impulse Responses

Figure 8 shows the daily impulse responses of the domestic business conditions index  $x_t^d$  to a one standard deviation shock to the respective prediction errors,  $v_t^j$ , where  $j$  captures

<sup>14</sup>From equation (2.10), it seems the external index impacts on the domestic business conditions index (through  $\phi$ ), but not vice versa. However, the Kalman filter partially allocates weights according to the covariance structure of the predicted state variance,  $\Sigma_{t|t-1}$  and matrix  $B$ .  $\phi$ ,  $\beta_{twi}^d$  and  $\beta_{tot}^d$  matter for calculating the weights for the external index—muting these two parameters will isolate the effects of the five domestic variables on the external index.

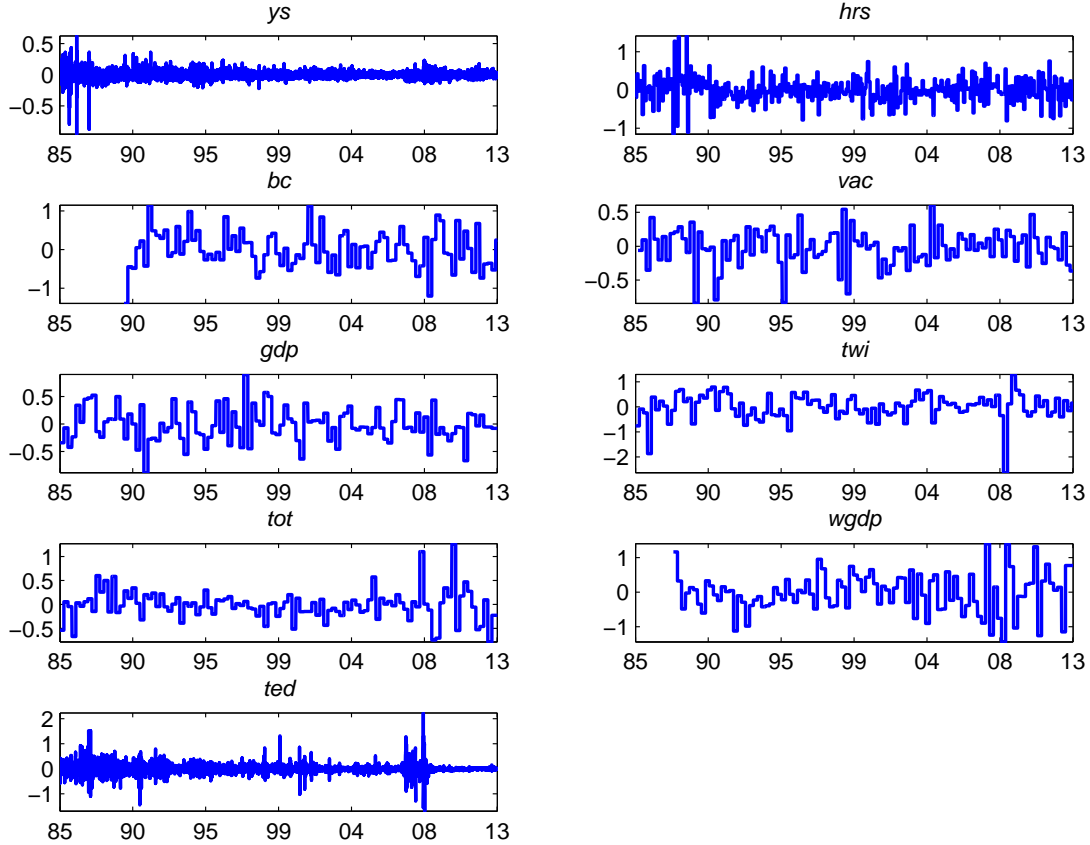


Figure 6: Interpolated prediction errors

the nine observable variables, assuming the evolution of the observable follows (2.2).<sup>15</sup> In each period, the Kalman filtered states need to be recomputed to construct the impulse responses.

Figure 9 shows the impact of a one standard deviation shock to the prediction error on each observable variable. All shocks are self-contained (i.e. only impact on themselves), except for the business confidence and terms of trade shocks. These have spill-over effects that arise from the updating procedure on the domestic business conditions index, which in turn drives other observable variables according to equation (2.2).

The shocks generally have a persistent impact on the domestic business conditions indicator (over more than 3 years), owing to the high value of the estimated daily AR(1) parameter  $\rho_d$ .<sup>16</sup>

<sup>15</sup>Since the observed data series are standardized with a zero mean and unit standard deviation, the observable vector is a  $9 \times 1$  zero vector in steady-state. For a temporary shock, it is observationally equivalent to consider a shock to either the respective prediction error or the measurement error. For example, a temporary shock to  $\eta^{ys}$  also creates an equal magnitude shock to the prediction error  $v_t^{ys}$  (see equation (2.8)), which updates the current domestic index according to (2.5). In the next period,  $yX_t$  is assumed to evolve according to (2.2), which means no prediction error (see (2.2) and (2.8)).

<sup>16</sup>Following a 1 per cent shock,  $\rho_d = 0.9975$  indicates 0.082 per cent remaining after 1000 days.

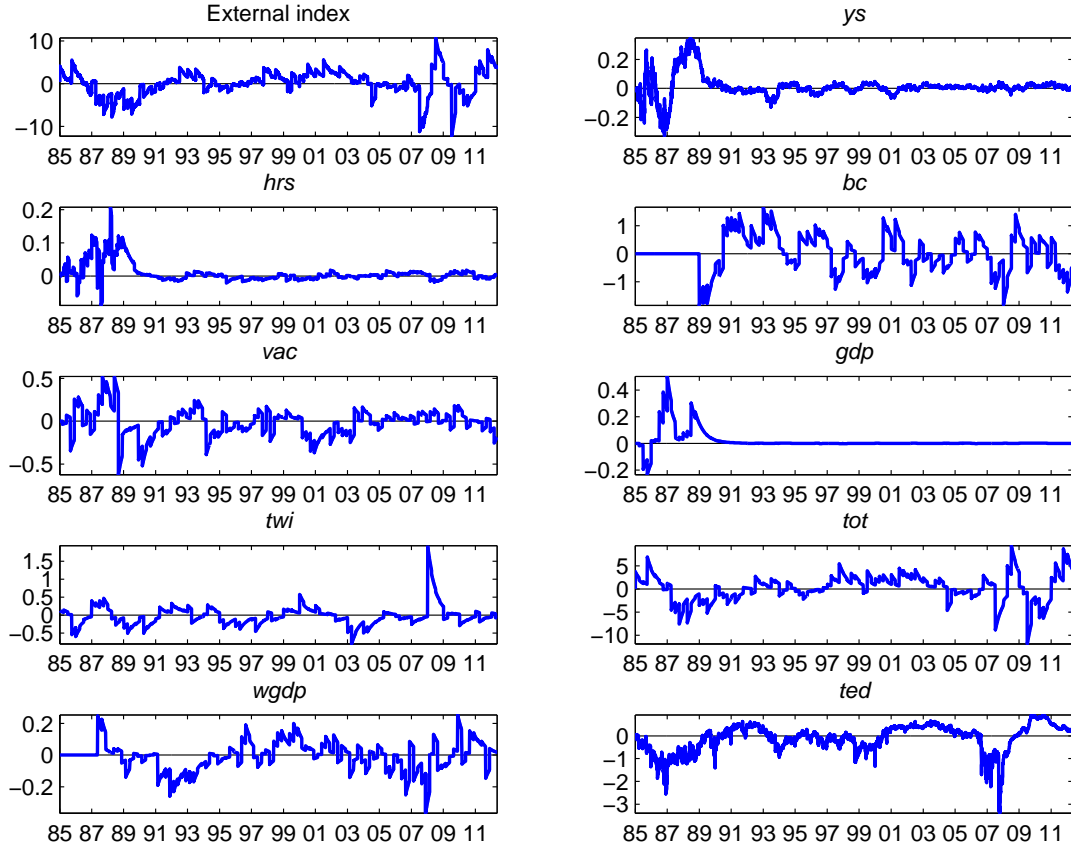


Figure 7: Accumulated period contributions of the prediction errors to the external index

Among all temporary unexpected shocks, only business confidence and the terms of trade shocks make a significantly large impact on business conditions in the Australian economy. A raised level of business confidence has an immediate self-fulfilling impact. It appreciates the real exchange rate, but this negative effect is not enough to change the large positive effect on vacancies. A stronger terms of trade gradually worsens domestic business conditions, peaking after 9 months. This occurs, in spite of the positive income effects from a terms of trade improvement, because the loss of international competitiveness amplified by real exchange rate appreciation dominates. Business confidence subsequently falls, and in turn vacancies.

Positive yield slope and TED spread shocks indicate a higher cost of raising longer term and riskier funds in financial markets, and thus negatively impact on the domestic business conditions indicator. Higher hours worked, job vacancies, business confidence and real GDP shocks indicate either current or future expected expansions of the real economy, and thus make positive contributions to the domestic index. World real GDP has a small but positive impact on the domestic business index, which peaks after about three quarters. An unexpected appreciation to the real trade-weighted index leads to an expansion of the domestic business index immediately, but this starts to deteriorate after about a month. This happens presumably due to the presence of a J-curve effect in Australia, possibly through staggered export and import contracts. Thus the Marshall-Lerner condition is likely only to begin to hold after at least a month and peaking after

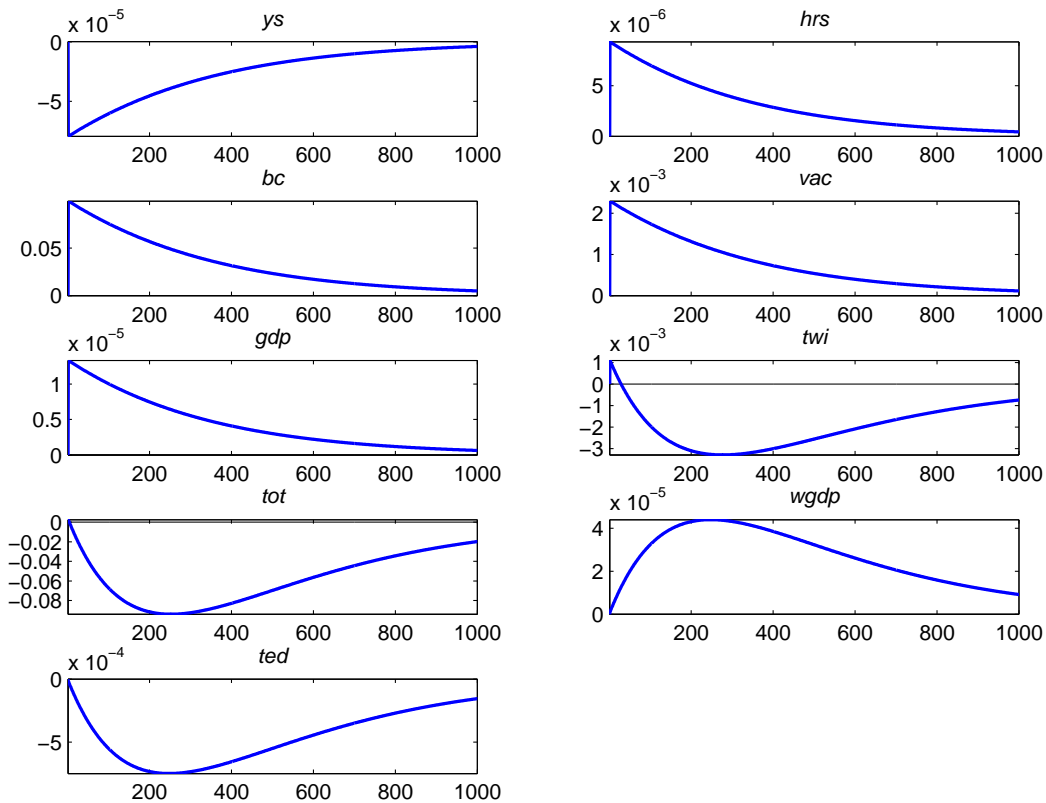


Figure 8: Impulse responses of the domestic business conditions index to 1 standard deviation prediction error shocks

a year.

### 3.3. The global financial crisis

Figure 10 focuses on the global financial crisis from 2007 to 2011. The top left subplot shows the estimated domestic business (blue solid line) and external (red dashed line) indices during this period.) The external index precedes the domestic business conditions index at the start of the global financial crisis. The external index fell dramatically in June 2008, while the domestic index followed suit 6 months later. The recovery of the external indicator from June 2008 to June 2009 was equally remarkable, and likely reflected the decline in the terms of trade and the resilience of China's growth, which played a major role in Australia's trade-weighted world GDP measure. Australia's domestic index recovered again with a lag of about 6 months. The external index fell again in mid-2010 due to the surging terms of trade, but this had a relatively small effect on Australia.

The other subplots of Figure 10 show the decomposition of the relative contributions of the prediction errors for each observed variable to the domestic and external indices from 2007 to 2011. This shows that the impact of the 2008 financial crisis on the domestic condition index was partially driven by the deteriorating TED spread, the terms of trade, and the sharp depreciation that worsened the current account in the immediate impact (before the Marshall-Lerner condition began to prevail). All of these further manifested in a wave of (forward-looking) business pessimism in late 2008 and thus an expected

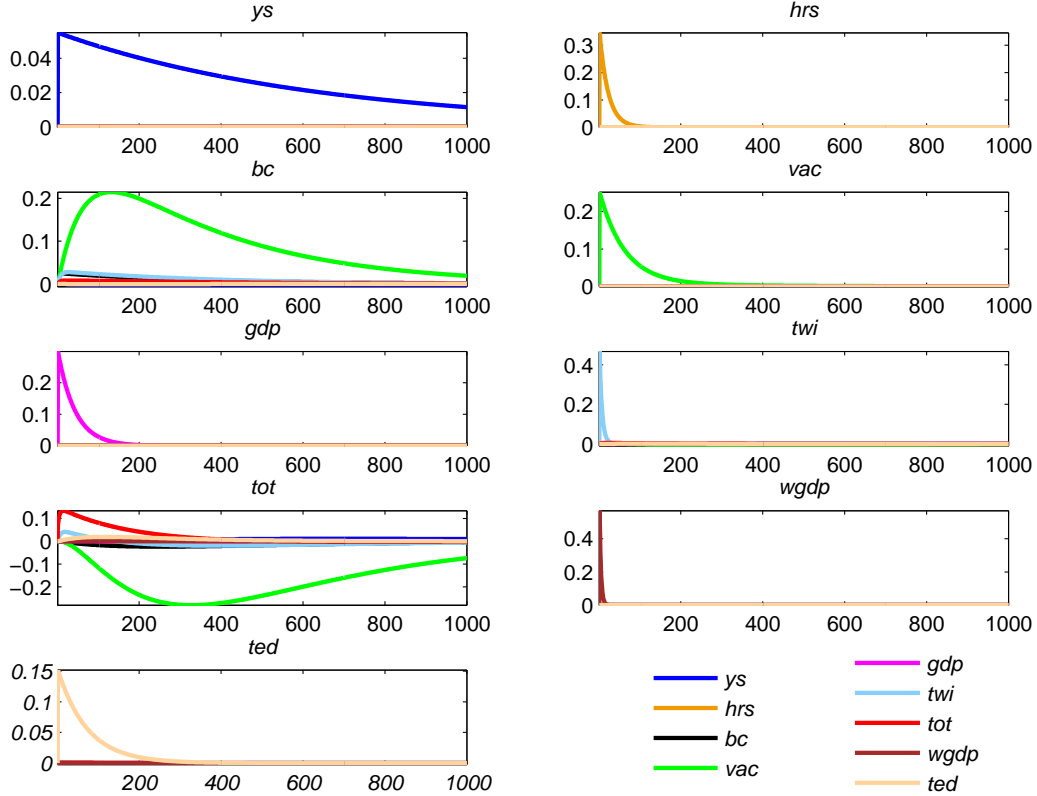


Figure 9: Impulse responses of the observable variables to 1 standard deviation prediction error shocks

deterioration in domestic business conditions in future periods. This was validated by the subsequent decline in actual hours worked, vacancies and GDP (see Appendix A).

#### 4. Conclusions

Using the Kalman filter in a dynamic factor model in state-space form, we have analyzed data on macroeconomic and financial variables arriving at different frequencies to estimate a domestic business conditions index and an external index for a small open economy, Australia.

These indices are useful because they summarize large amounts of information in an efficient way. Looking behind the indices, we find a number of important conclusions.

First, a survey-based measure of business confidence turns out to play a vital role in the domestic business conditions index. By its construct, this observed measure is forward-looking, and thus it has predictive content for other variables so that shocks to it tend to have subsequent spill-over effects that affect the real economy in a partially self-fulfilling way. The Kalman filter gives prominence to this variable because the filter optimizes on predictive content. Thus our estimated domestic business conditions index can be considered a leading indicator of Australian economic activity.

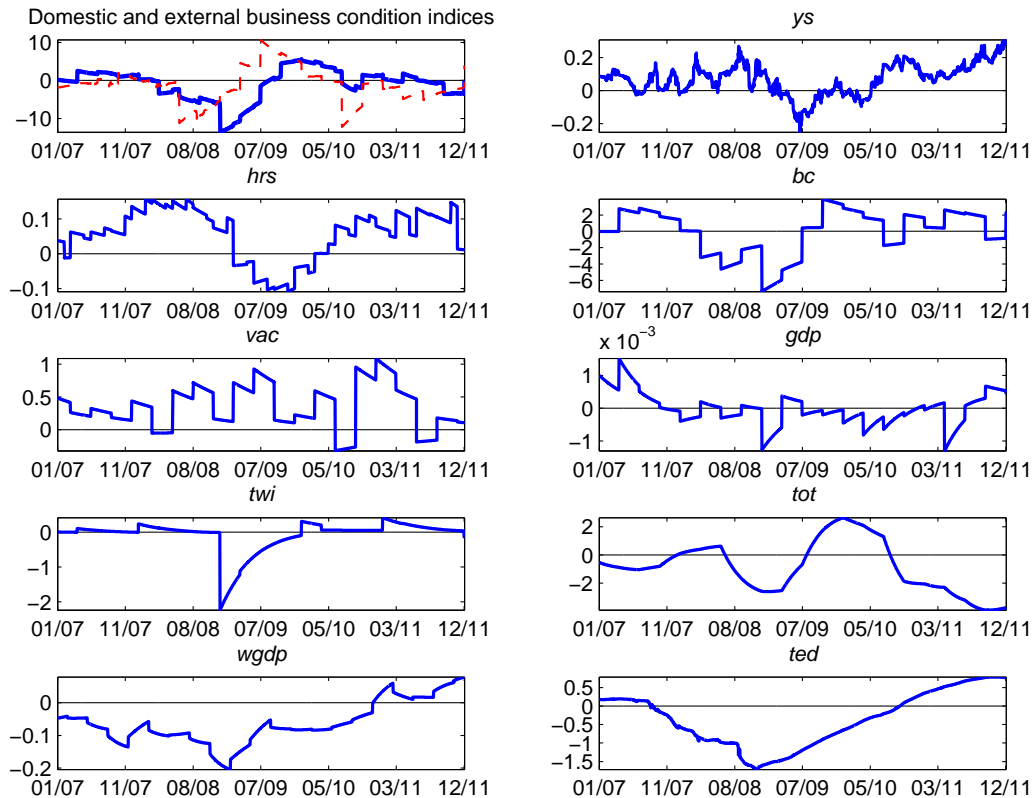


Figure 10: The domestic business conditions indices with accumulated period contributions from 2007 – 2011

Second, the terms of trade is a critical driver for the external index, which is what one would expect for a small open economy. Again, the Kalman filter favours this variable because it has relatively high predictive content for other variables. It is directly related to domestic conditions, as well as indirectly through the lagged effect of the external index on the domestic index.

Third, financial variables (the domestic yield spread and the external TED spread) only appear to play an important role in episodes of large recessions or crises.

Fourth, Australia's experience of the global financial crisis of 2008 was fundamentally driven by the external index, which then fed into deteriorating business confidence. Australia had a short-lived and mild crisis, and recovered quickly. In 2013, the external index was positive and improving, yet the domestic index was languishing, which may be because of undue pessimism in business confidence.

In further work, we will extend the range of observable variables used. We will try to uncover other possible financial drivers for the real economy, for example stock market and credit market measures. We will look at measures of household confidence which may complement the business confidence variable, and we will aim to include explicit measures of the stance of monetary and fiscal policy.

## Appendix A. Data series

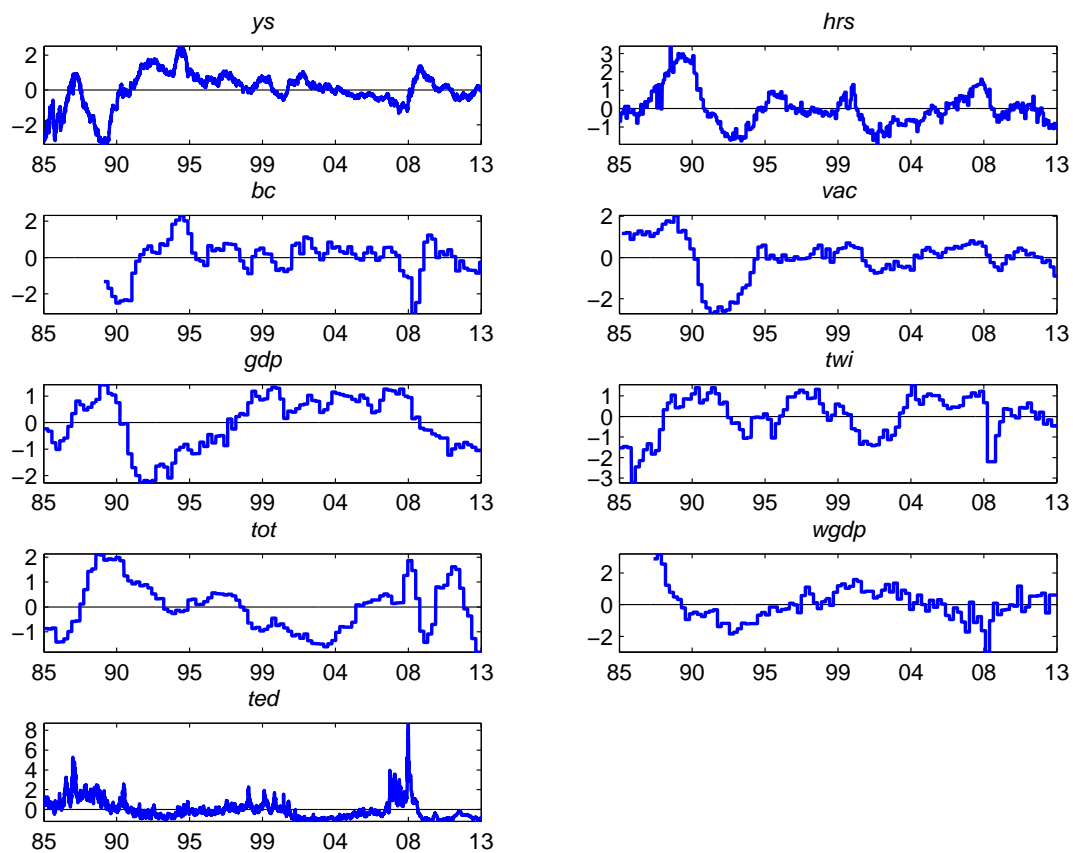


Figure A.11: Transformed observable variables



## Appendix B. Time detrending

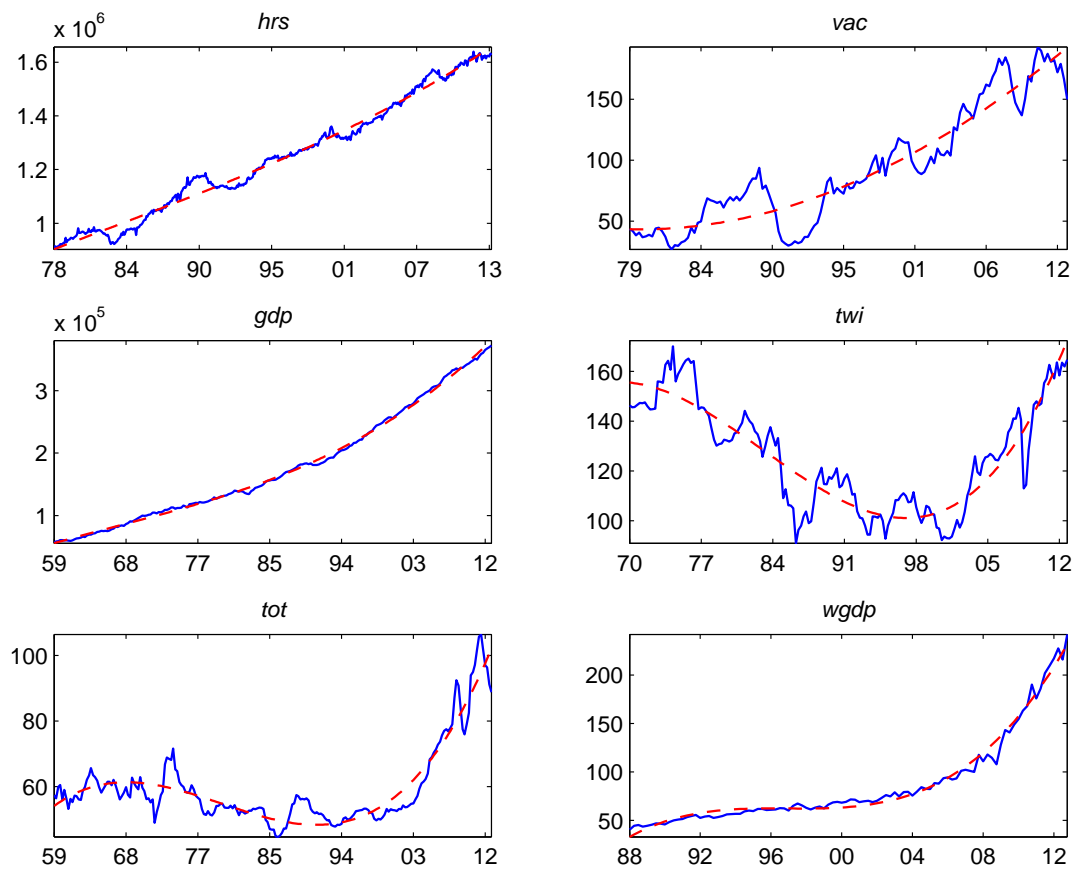


Figure B.12: Original series and time trends

## Appendix C. The role of the size of the estimated measurement error variances

This appendix disentangles the impact of the relative size of the estimated measurement error variances on the information updating by the Kalman filter. To illustrate the idea, we ignore the external aspects of our model, and only consider two stock variables: business confidence and vacancies, and consider what happens when the variance of the measurement error of the former approaches zero. Since the AR(1) processes in the measurement equation will not alter our conclusions, we drop them also. The measurement and state equation become:

$$X_t^d = \rho_d X_{t-1}^d + \epsilon_t^d \quad (\text{C.1})$$

$$\begin{bmatrix} bc_t \\ vac_t \end{bmatrix} = \begin{bmatrix} \beta_{bc} \\ \beta_{vac} \end{bmatrix} X_t^d + \begin{bmatrix} \eta_t^{bc} \\ \eta_t^{vac} \end{bmatrix} \quad (\text{C.2})$$

where both  $\eta^{bc}$  and  $\eta^{vac}$  have a 0 mean and respective variances,  $\sigma_{bc}^2$  and  $\sigma_{vac}^2$ . Denote  $\hat{X}_{t-1}^d$  and  $\hat{\Sigma}_{t-1}$  as the estimated mean and variance for  $X^d$  using information up to time  $t-1$  and define the determinant of the variance-covariance matrix of prediction error  $v_t$  as  $\chi = \left( \rho_d^2 \hat{\Sigma}_{t-1} + \sigma_d^2 \right) (\beta_{bc}^2 \sigma_{vac}^2 + \beta_{vac}^2 \sigma_{bc}^2) + \sigma_{bc}^2 \sigma_{vac}^2$ , which satisfies  $\chi > 0$ . The Kalman gain matrix, prediction error, estimated state and variance for time  $t$  are:

$$K_t = \frac{1}{\chi} \begin{bmatrix} \beta_{bc} \sigma_{vac}^2 \left( \hat{\Sigma}_{t-1} \rho_d^2 + \sigma_d^2 \right) & \beta_{vac} \sigma_{bc}^2 \left( \hat{\Sigma}_{t-1} \rho_d^2 + \sigma_d^2 \right) \end{bmatrix} \quad (\text{C.3})$$

$$v_t = \begin{bmatrix} bc_t - \beta_{bc} \rho_d \hat{X}_{t-1}^d & vac_t - \beta_{vac} \rho_d \hat{X}_{t-1}^d \end{bmatrix}' \quad (\text{C.4})$$

$$\hat{X}_t^d = \frac{\sigma_{bc}^2 \sigma_{vac}^2 \rho_d \hat{X}_{t-1}^d + \left( \hat{\Sigma}_{t-1} \rho_d^2 + \sigma_d^2 \right) (\beta_{bc} \sigma_{vac}^2 bc_t + \beta_{vac} \sigma_{bc}^2 vac_t)}{\chi} \quad (\text{C.5})$$

$$\hat{\Sigma}_t = \frac{\sigma_{bc}^2 \sigma_{vac}^2 \left( \hat{\Sigma}_{t-1} \rho_d^2 + \sigma_d^2 \right)}{\chi} \quad (\text{C.6})$$

The Kalman gain matrix measures the relative contribution or weight of the prediction errors to the estimated index,  $\hat{X}_t^d$ . From equation (C.3), note that the Kalman weights depend crucially on the variance of the measurement errors, and the signs of these weights depend on the respective  $\beta$ s. From equation (C.5), the contributions of each observable depends positively on its own  $\beta$  and on the other observable's variance. Suppose the point estimate of  $\sigma_{bc}$  is significantly smaller than  $\sigma_{vac}$ , (and possibly not statistically significant from 0), and then take the limiting case of  $\sigma_{bc} = 0$ :

$$K_t = \begin{bmatrix} \frac{1}{\beta_{bc}} & 0 \end{bmatrix} \quad \hat{X}_t^d = \frac{bc_t}{\beta_{bc}} \quad \hat{\Sigma}_t = 0$$

In this case, the Kalman filter puts no weight on  $vac_t$  and updates the states in proportion to  $bc_t$ .

The intuition can be seen from equation (C.2), which shows that the fluctuations of  $bc_t$  are either due to the measurement error  $\eta^{bc}$  or the index  $X_t^d$ . With a 0 variance of its measurement error, its fluctuations are reflected solely by the index. Therefore, the index is updated solely by the business confidence variable. Note that the estimated state variance of  $\hat{\Sigma}_t$  is 0, indicating the state is updated with certainty.

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