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Managing Risks from Climate Impacted Hazards - The Value of Investment Flexibility under Uncertainty

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Chi Truong, Stefan Trueck and Supriya Mathew

Managing Risks from Climate Impacted Hazards - The Value of Investment Flexibility under Uncertainty $\stackrel{\bigstar}{\Rightarrow}$

Chi Truong^a, Stefan Trück^a, Supriya Mathew^b

^aFaculty of Business and Economics, Macquarie University, NSW, Australia, 2109 ^bNorthern Institute, Charles Darwin University, NT, Australia, 0909

Abstract

Incomplete knowledge about climate change and the related uncertainty in climate prediction makes adaptation inherently difficult. We introduce a real options framework to determine optimal adaptation to catastrophic risk that takes into account climate change uncertainty. The framework can be used to select optimal adaptation investments from a number of alternatives or to determine the optimal investment sequence from available projects. Using a case study of bushfire risk management, we illustrate that the proposed framework can significantly increase the value of investments. While the results are found to be less impacted by the uncertainty parameter, they are quite sensitive to the expected extent of climatic change. This implies that for the purpose of investment analysis under climate change uncertainty, it is important to include as many climate change predictions as possible, in particular if a higher number of scenarios increases the accuracy and robustness of the forecasts. Investment cost is found to have a large impact on the loss caused by the NPV rule, suggesting that the real options framework is more important for high sunk cost projects.

Keywords: Climate change adaptation, Investment under Uncertainty, Catastrophic risk, Real option

JEL Codes: D81, Q54, C61, H54.

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Email addresses: chi.truong@mq.edu.au (Chi Truong), stefan.trueck@mq.edu.au (Stefan Trück), Supriya.Mathew@cdu.edu.au (Supriya Mathew) Preprint submitted to Elsevier July 31

1. Introduction

Climate induced catastrophes including floods, storm surges and bushfires, are predicted to occur more frequently and cause more severe damages in the coming years under the impact of climate change. With a higher global temperature, the climate system is more energetic, and catastrophes become more likely to occur (Solomon, 2007). The increased frequency and severity of natural disasters in recent decades have made such a future outlook particularly concerning and serious attention has been paid to climate change adaptation that mitigates catastrophic risks (Van Aalst, 2006).

Optimal adaptation to climate change, however, requires input from all levels of government and could potentially be one of the most challenging tasks in environmental management. While it has often been argued that adaptation action may be most effective at the local level, local stakeholders are confronted with the complex and problematic task of planning and implementing mitigation and adaptation actions within existing budget constraints. Therefore, optimal planning requires an appropriate economic framework to evaluate potential climate change adaptation options, and to justify the implemented actions.

To determine optimal strategies for catastrophic risk mitigation, several issues need to be dealt with. First, a risk quantification framework is required that can incorporate increasing losses induced by economic development in the region as well as the growth of loss frequency and severity due to climatic change. Second, the framework needs to quantify the magnitude of the uncertainty related to climate change impact so that the impact of uncertainty can be incorporated into the investment decision. With respect to this second issue, the usual approach of using historical climate data to estimate climate change uncertainty may not be satisfactory. The change in the climate system occurs slowly, with significant lags from the time when emission is admitted into the atmosphere. Therefore using historical climate data to predict climate change may result in conservative estimates that do not reflect the impacts of new and recent emission or feedback mechanisms. Third, with uncertainty about the extent to which the climate will change, and the irreversibility of investment projects, the opportunity to invest in an adaptation project is analogous to a financial call option, and optimal investment needs to take into account the investment option value. In addition, there are usually several adaptation projects that could be invested and the issues of which investment project to select as well as the optimal sequence of adaptation investment, i.e. which projects to be invested first and which ones to invest later, need to be dealt with.

The problem of evaluating catastrophic loss reduction investment under the impacts of climate change has been examined in previous studies. These include the work of West et al. (2001); Michael (2007); Kirshen et al. (2008); Tsvetanov and Shah (2013) who examine storm surge risk in coastal areas as well as studies by Brouwer and van Ek (2004); Zhu et al. (2007); Bouwer et al. (2010); Mathew et al. (2012) who examine flood risk in coastal and riverine regions and Truong and Trück (2016a) who examine the problem of bushfire risk management in an urban area.

West et al. (2001) evaluate the increased damage of storm surge in a hypothetical region under the impact of sea level rise. They use the loss distribution approach (LDA) to model house losses and insurance premium data to estimate the parameters of the model. Brouwer and van Ek (2004) evaluate flood control policies in the Netherlands, taking into account ecological benefits. Michael (2007) evaluates the increased damage of storm surge in Maryland under climatic change, using the reduction in house elevation under increased sea levels to determine the additional cost of insurance premiums attributed to sea level rise. Zhu et al. (2007) determine the optimal levee height and setback for a floodplain in California, using dynamic programming to maximize the total reduction in the expected loss.

Kirshen et al. (2008) evaluate the increased damage of storm surge in Boston when the sea level rises, taking into account adaptation responses. They construct various scenarios that differ in the number of storm surges, the extent of sea level rise and the level of adaptation, and then evaluate the expected damage in each scenario. Bouwer et al. (2010) evaluate the potential damage from river flooding in the Netherlands. They estimate the increase in the frequency of flood events under climate change based on the increase in the cumulated 10 days rainfall and estimate flood damage using a damage

scanner model that relates flood damage to flood depth. Mathew et al. (2012) evaluate adaptation options to reduce flood risk in India using the LDA. Loss frequency and severity distributions are first estimated using expert opinions and then updated using loss observations in the study region via the Bayes' rule. Adaptation measures are ranked by the net present value (NPV) rule as well as the triple bottom line that incorporates environmental benefits. Tsvetanov and Shah (2013) evaluate the optimal investment time for adopting protection against storm surge in Connecticut. They use the HAZUS-MH MR4 risk assessment software provided by the Federal Emergency Management Agency to estimate the damage-return period curve for each time period, which is then used to estimate the expected damage. The optimal adaptation time is selected to maximize the NPV of adaptation. Truong and Trück (2016a) examine the impact of risk aversion and optimal investment timing on the value of an investment project that reduces bushfire risk in an urban area. They extend the standard LDA to allow for the impacts of increased loss due to economic growth and higher frequency of bushfires as a result of climate change. The loss frequency distribution is calibrated based on projections of previous climate change impact studies and the loss severity distribution is estimated based on the average number of houses damaged in a fire event and house construction cost.

In most of the previous studies, the NPV rule is used to determine the investment decision: a project is invested if its NPV is positive. The NPV rule does not consider the possibility that the project can be invested at some future time. West et al. (2001), Zhu et al. (2007) and Truong and Trück (2016a) departed from the NPV rule to examine the optimal time to invest. In these frameworks, the optimal time to invest is the time when the NPV of the project is highest compared to all other investment times. However, none of these studies consider the value of investment flexibility that helps to cope with the uncertainty of climate change.

Flexibility in decision making has been widely recognised to play an important role in adaptation under climate change uncertainty. In recent years, policy makers have suggested to follow adaptation plans that 'maximize flexibility, keep options open and avoid lock-in' Kuijken (2010). Several methods including 'adaptive policymaking' (Kwakkel and Haasnoot, 2012), 'adaptation pathways' (Haasnoot et al., 2012), 'dynamic adaptive policy pathways' (Haasnoot et al., 2013) and 'robust adaptive policy' (Lempert and Groves, 2010) have been proposed to obtain the objectives of maximizing flexibility and keeping options open. These approaches are, however, without criticisms (Mills et al., 2014; Truong and Trück, 2016b). By maximizing flexibility regardless of the opportunity cost, the benefit of durable investment and long lasting policies will be foregone. As suggested by Mills et al. (2014), the price of maximizing flexibility at any cost can be quite high. Our paper provides a method to consider adaptation pathways - which in the following we will refer to as investment sequence - with the important feature that we take into account both the benefit and the cost of flexibility.

In this paper, we introduce an economic framework to select optimal adaptation investments to catastrophic risk that takes into account the uncertainty of climate change. To evaluate investment projects, we adopt a loss distribution approach and a doubly stochastic Poisson process that allows for stochastic growth of the losses. A doubly stochastic Poisson process has also been proposed for the pricing of catastrophic bonds, see, e.g. Lin et al. (2009). We also provide a real options framework that allows for selection of an investment project from several projects as well as for the management of sequential investment into different projects. Although the problem of optimally selecting a project from many alternatives has been examined by Décamps et al. (2006), their discussion is quite technical and may be inaccessible to many.

Our model is developed in a continuous time framework and is simple to implement. Using a case study of bushfire risk management, we illustrate that the consideration of investment flexibility can significantly increase the value of adaptation investment, above the current NPV. In addition, the investment value can be further increased by considering the optimal sequence of projects to be invested. In contrast, ignoring investment options and investment sequencing can result in the elimination of investment options that are valuable for the management of future risk. In conducting sensitivity analysis, we find that with more serious climate change, lower discount rates and/or lower investment costs, the loss due to adopting the suboptimal NPV rule is lower for projects that already have a positive NPV. Furthermore, the results on managing sequential investment suggest that it is optimal to invest in a low sunk cost project first and preserve the flexibility of investing in a high sunk cost project for the possible case when the catastrophic risk is higher. This result is also consistent with findings provided by earlier studies that are more focused on the qualitative analysis of adaptation strategies, see e.g. Hallegatte (2009).

Our results also suggest that the option value and investment decisions are relatively insensitive to the uncertainty parameter, while they are quite sensitive to the expected extent of climate change impacts and to the applied discount rate. This implies that for the purpose of investment analysis under the uncertainty of climatic change, it is important to include as many climate impact predictions as possible, in particular when a higher number of predictions increases the accuracy and robustness of the expected forecasts. Such expansion of the set of climate forecasts will increase the robustness of the statistical model within the investment framework without compromising the objective of maximising the investment value. Investment cost is also found to have a large impact on the loss due to the use of the NPV rule, which implies that the real options framework is more important for high sunk cost projects.

The remainder of the paper is organized as follows. Section 2 outlines and analyzes the developed modeling framework. Section 3 provides an application of the framework in a case study, using catastrophic risks from bushfires as an empirical example. The section also examines the impacts of optimal investment for alternative investment versus sequential investment, investment costs, and the applied discount rate on the results. Section 4 concludes and provides suggestions for future research.

2. Modeling Framework

We adapt the standard LDA to quantify potential losses from extreme events, taking into account the growth of loss severity and the uncertain increase of the frequency of events under climate change. We then analyze investment models to select one project from several alternatives and to determine the optimal sequence of projects when several projects can be invested.

2.1. Frequency and Severity of Climate Impacted Hazards

The LDA is commonly used to model catastrophic losses in the insurance and banking sector as well as losses arising from operational risks, see e.g., Klugman et al. (2008); Shevchenko and Wüthrich (2006). There are also a few applications of the framework to modeling losses related to natural or climate impacted hazards such as, e.g., storms, earthquakes, flooding and bushfire (West et al., 2001; Härdle and Cabrera, 2010; Mathew et al., 2012; Truong and Trück, 2016a). With this approach, the total loss over a period (0, t] is modeled as a compound Poisson process:

$$S_t = \sum_{n=1}^{N(t)} X_n,$$
 (2.1)

where N(t) denotes the number of catastrophic events occurring from time 0 up to time t, and X_n is the loss caused by the n^{th} event. In this standard model, N(t) is assumed to follow a homogeneous Poisson process with intensity $\Lambda > 0$, X_n is assumed to be independently and identically distributed according to a distribution H(X) and X_n is independent from N(t). A realization of two catastrophic events with severities x_1, x_2 over period (0, t] corresponds to $\{N(t) = 2; X_1 = x_1, X_2 = x_2\}$.

The standard model (2.1) can be extended to incorporate growing loss severity and frequency. We allow the loss severity to grow over time by modeling the catastrophic loss X_n as a product of the catastrophic loss under zero growth X_0 and a growth component:

$$X_n = X_0 e^{\gamma \tau_n}.\tag{2.2}$$

In Equation (2.2), γ is the growth rate of the risk prone asset values, and τ_n is the random time when the n^{th} climate impacted event occurs, which is determined by the Poisson process. A growth in the value of risk prone assets may be due to, for example, increases in the number of properties in a region or investment in additional infrastructure. It may also be a result of improvements in properties' or assets' conditions as the economy grows.¹ The random variable X_0 is the catastrophic loss when the values of the assets at

¹The exponential growth of loss severity is consistent with the pattern of natural disaster losses observed in Australia, see, e.g., Crompton et al. (2006); Crompton and McAneney (2008), and the pattern of flood losses in Netherlands, see, e.g., Brouwer and van Ek (2004).

risk do not grow over time, which can also be interpreted as a measure of the destruction force of the climate impacted hazard. It is assumed that X_0 is identically, independently distributed and X_0 is independent from N(t) and therefore τ_n . In the following, we denote the expected value of X_0 by β .

Previous climate change adaptation studies, such as Fisher and Rubio (1997), Gersonius et al. (2013), have used Geometric Brownian Motion (GBM) processes to model uncertain development in climate variables. As a result of stochastic variation in climate variables, the frequency of catastrophic events will also vary stochastically, since the occurrence of catastrophic events are closely linked to climate variables (Lucas, 2010). As such, we assume that the number of catastrophic events N(t) that occur over period (0, t]follows a doubly stochastic Poisson process, with the intensity $\Lambda(t)$ of the process evolving according to a Geometric Brownian Motion (GBM):

$$d\Lambda_t / \Lambda_t = \mu dt + \sigma dW_t, \tag{2.3}$$

where W_t is a Wiener process, μ is the expected growth rate of Λ_t and σ represents the magnitude of the uncertainty in predicting future values of Λ_t .

2.2. Choosing among Alternative Investment Projects

In this paper, we assume that the decision maker is risk neutral in order to focus our analysis on the value of flexibility. For a study that incorporates risk aversion into the analysis of optimal adaptation investments, although in a simpler setting, we refer to Truong and Trück (2016a). Incorporating risk aversion will increase the value of projects as well as the value of the option to invest and will therefore typically lead to earlier investment in adaptation projects.

Let us now assume that the decision maker can invest in one of n projects only. Each project is assumed to last infinitely and the investment cost is sunk once committed. This is a standard assumption and has been adopted in other real options studies (Dixit and Pindyck, 1994; Fisher, 2000; Pindyck, 2002; Baranzini et al., 2003; Gollier and Treich, 2003). In empirical applications, an infinitely lasting project is constructed as a series of finite lifetime projects, where a new finite lifetime project is put in place whenever the previous project is fully depreciated.

We assume that each project *i* has investments cost I_i , a maintenance cost flow C_i and reduces the loss frequency by a proportion k_i , i = 1, ..., n. This means that given the Poisson intensity Λ_s at time *s*, project *i* reduces the flow of expected loss at time *s*, $\beta e^{\gamma s} \Lambda_s$, by a proportion k_i . At time *t*, if the decision maker decides to invest in project *i*, he/she will obtain the expected NPV of project *i* given by:

$$V_i(\Lambda_t) = E\left[\int_t^\infty e^{-rs} [k_i \beta e^{\gamma s} \Lambda_s - C_i] ds - e^{-rt} I_i |\Lambda_t\right], \qquad (2.4)$$

where r is the discount rate. After using the expectation of Λ_s , $E[\Lambda_s|\Lambda_t] = \Lambda_t e^{\mu(s-t)}$, where s > t, and integrating the expression, the value of project *i* invested at time *t*, when the Poisson intensity assumes a value Λ_t , is given by

$$V_i(\Lambda_t) = \frac{k_i \beta \Lambda_t}{r - \mu - \gamma} e^{-(r - \gamma)t} - e^{-rt} (I_i + C_i/r).$$
(2.5)

The investment problem is that at any time t, the decision maker observes the value Λ_t and determines whether to invest in one of the projects i = 1, ..., n or to defer the investment to a later time. If the decision maker decides to invest in project i, he/she gets the value of project i, $V_i(\Lambda_t)$, and the decision process stops. If the decision maker decides to wait, then at a later point in time $t + \Delta t$, he/she can consider the decision of whether to invest in one of the projects or to defer the investment again. As a result, the value of the option to invest is the maximum of the values of the individual projects and the value of deferring the investment. In other words, if $F(\Lambda_t)$ is the value of the option to end the n projects, then $F(\Lambda_t)$ satisfies:

$$F(\Lambda_t) = \max\left\{V_1(\Lambda_t), V_2(\Lambda_t), ..., V_n(\Lambda_t), e^{-r\Delta t} F(\Lambda_{t+\Delta t})\right\}.$$
(2.6)

Figure 1 illustrates the situation, where a dominant project exists (*Panel a*) as well as the situation where different projects yield the highest NPV for different values of the intensity parameter Λ (*Panel b*). When a dominant project *m* exists, i.e. there is a project whose NPV is higher than the positive NPV of all other projects for any choice



Figure 1: NPV profiles of different projects. A project with a NPV always higher than the positive NPV of all other projects (for any choice of Λ) is a dominant project. For example, the project in *Panel a*, where the NPV is represented by the dotted line is a dominant project. Panel b illustrates the situation where no dominant project exists. Different projects yield the highest NPV for different ranges of the intensity parameter Λ.

of Λ , Equation (2.6) reduces to

$$F(\Lambda_t) = \max\left\{V_m(\Lambda_t), e^{-r\Delta t}F(\Lambda_{t+\Delta t})\right\},\tag{2.7}$$

and the option to invest in one of the *n* projects reduces to the option to invest in project m.

Applying Ito's Lemma and subtracting $F(\Lambda_t)$ on both sides of the equation then yields the stochastic differential equation:

$$0 = \max\left[V_m(\Lambda_t) - F(\Lambda_t), 0.5\sigma^2 \Lambda_t^2 F_{\Lambda\Lambda} + \mu \Lambda_t F_{\Lambda} - rF\right].$$
(2.8)

The value of the option is then given by

$$F(\Lambda_t) = B_m \Lambda_t^{\alpha}, \tag{2.9}$$

where $\alpha = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$ and $B_m = e^{-rt} \left(\frac{\beta e^{\gamma t}}{r - \mu - \gamma}\right)^{\alpha} \frac{(\alpha - 1)^{\alpha - 1}}{\alpha^{\alpha}} \frac{k_m^{\alpha}}{(I_m + C_m/r)^{\alpha - 1}}$. The

option is exercised when Λ_t is greater or equal to the investment threshold given by:

$$\Lambda_m^* = (I_m + C_m/r)[\alpha/(\alpha - 1)] \frac{r - \mu - \gamma}{k_m \beta e^{\gamma t}}.$$
(2.10)

When no dominant project exists as illustrated in *Panel b* of Figure 1, the optimal investment rule may include investing in a non-dominated project when the state variable Λ_t is in a low range and investing in another non-dominated project when the state variable is in a high range. Suppose that there are two non-dominated projects, where Project 1 is dominant for a lower range and Project 2 dominates for a higher range of values for the state variable Λ_t . For a value of Λ_t close to zero, the optimal decision is not to invest in any of the projects. For such a scenario, since the optimal decision is to wait, the value of the option to invest is given by:

$$F(\Lambda_t) = B\Lambda_t^{\alpha},\tag{2.11}$$

where α is as in (2.9) and B is a parameter to be determined. Since the decision maker can choose to invest in either of the projects at any time, the option to invest in one of the two projects is the higher of the values of the option to invest in each individual projects, $F_1(\Lambda_t)$ and $F_2(\Lambda_t)$. This is satisfied if $B = \max_i B_i$, $i \in \{1, 2\}$, where $B_i = e^{-rt} \left(\frac{\beta e^{\gamma t}}{r-\mu-\gamma}\right)^{\alpha} \frac{(\alpha-1)^{\alpha-1}}{\alpha^{\alpha}} \frac{k_i^{\alpha}}{(I_i+C_i/r)^{\alpha-1}}.$

If $B_2 > B_1$ or equivalently,

$$\frac{k_2^{\alpha}}{(I_2 + C_2/r)^{\alpha - 1}} > \frac{k_1^{\alpha}}{(I_1 + C_1/r)^{\alpha - 1}},$$
(2.12)

then the value of the option to invest in alternative projects, $F(\Lambda_t)$, is equal to the value of the option to invest in Project 2, $F_2(\Lambda_t)$. The optimal decision is to wait for investment in Project 2 while the state variable Λ_t is lower than Λ_2^* (the investment threshold for Project 2) and to invest in Project 2 when Λ_t is higher than Λ_2^* . Thus, in this case, the option to invest in one of the two projects is the same as the option to invest in Project 2.

On the other hand, if $B_2 < B_1$, then a low value for Λ_t yields $F_1(\Lambda_t) > F_2(\Lambda_t)$ and the value of the option is equal to the value of the option to invest in Project 1. It is optimal

to wait while Λ_t is lower than the optimal investment threshold of Project 1, Λ_1^* . When Λ_t is higher or equal to Λ_1^* , and lower than a level $\hat{\Lambda}$, it is optimal to invest in Project 1 immediately. When Λ_t is above $\hat{\Lambda}$ and lower than the optimal investment threshold for Project 2, Λ_2^* , it is optimal to wait for investing in Project 2. For Λ_t above Λ_2^* , immediate investment in Project 2 is optimal. The only difference between this problem and the problem of investing in an individual project is that we need to determine $\hat{\Lambda}$.

Note that at $\hat{\Lambda}$, the decision maker is indifferent between immediate investment in Project 1 and waiting for investment in Project 2, i.e. $V_1(\hat{\Lambda}) = F_2(\hat{\Lambda})$, and therefore $\hat{\Lambda}$ can be found by solving equation:

$$\frac{k_1 \beta \hat{\Lambda}}{r - \mu - \gamma} e^{-(r - \gamma)t} - e^{-rt} (I_1 + C_1/r) = B_2 \hat{\Lambda}^{\alpha}.$$
(2.13)

The value of the option to invest in alternative projects is equal to the value of the option to invest in Project 1 when Λ_t is lower than Λ_1^* ; equal to the value of Project 1 when Λ_t is between Λ_1^* and $\hat{\Lambda}$; equal to the value of the option to invest in Project 2 when Λ_t is between $\hat{\Lambda}$ and Λ_2^* and equal to the value of Project 2 when Λ_t is above Λ_2^* .

2.3. Sequential Investment

So far in the considered setting with alternative investment projects, although we consider many projects at the same time, we have restricted the decision maker to invest in only one project over the whole time horizon. However, this would only be the case if the decision maker is subject to a restricted budget and can afford to invest in only one project. When the budget constraint is relaxed, it may be optimal to invest in an additional project at a later point in time after the initial investment, if the state variable Λ_t increases to a sufficiently high level. Sequential investment results in a higher option value compared to alternative investment due to more investment opportunities. For simplicity and to illustrate the approach, in the following we consider two projects only in the case of sequential investment. However, an extension of the approach to sequential investment with a higher number of projects is straightforward.

Suppose that Project 1 is invested first, and when Λ is sufficiently high, Project 2 is

invested. The investment problem is:

$$F_{1}(\Lambda_{t}) + F_{12}(\Lambda_{t}) = \max_{\tau_{1},\tau_{2}} E\left[\int_{\tau_{1}}^{\infty} [k_{1}\beta e^{\gamma s}\Lambda_{s} - C_{1}]ds - e^{-r\tau_{1}}I_{1} + \int_{\tau_{2}}^{\infty} [k_{2}(1-k_{1})\beta e^{\gamma s}\Lambda_{s} - C_{2}]ds - e^{-r\tau_{2}}I_{2}|\Lambda_{t}\right],$$
(2.14)

where $F_{12}(\Lambda_t)$ is the value of the option to invest in Project 2, after Project 1 has already been invested. Similarly, if Project 2 is invested first, then $F_2(\Lambda_t) + F_{21}(\Lambda_t)$ is obtained. In considering which project to be invested first, the decision maker needs to select a sequence that maximizes the option to invest, $F_s(\Lambda_t)$, i.e.

$$F_s(\Lambda_t) = \max\{F_1(\Lambda_t) + F_{12}(\Lambda_t), F_2(\Lambda_t) + F_{21}(\Lambda_t)\}.$$
(2.15)

At a value Λ_t close to zero, the values of the options are as in (2.9), with $k_{12} = k_2(1-k_1)$ and $k_{21} = k_1(1-k_2)$. From the expression of option values, it is optimal to invest in Project 1 first, if:

$$\frac{k_1^{\alpha}}{(I_1 + C_1/r)^{\alpha - 1}} + \frac{((1 - k_1)k_2)^{\alpha}}{(I_2 + C_2/r)^{\alpha - 1}} > \frac{k_2^{\alpha}}{(I_2 + C_2/r)^{\alpha - 1}} + \frac{((1 - k_2)k_1)^{\alpha}}{(I_1 + C_1/r)^{\alpha - 1}}.$$
 (2.16)

After the optimal sequence of investments has been identified, investments are conducted using the optimal investment thresholds:

$$\Lambda_i^* = (I_i + C_i/r)[\alpha/(\alpha - 1)] \frac{r - \mu - \gamma}{k_i \beta e^{\gamma t}}, \qquad (2.17)$$

where $i \in \{1, 2, 12, 21\}$ and $I_{12} = I_2, I_{21} = I_1, C_{12} = C_2, C_{21} = C_1, k_{12} = (1 - k_1)k_2, k_{21} = (1 - k_2)k_1.$

Clearly, the illustrated framework for sequential investment can easily be extended to a situation where the optimal sequence of investments involves more than two projects.

2.4. Loss due to using the NPV rule

Consider an investment project, for example, the investment project m in section 2.2 with the real options value $F(\Lambda_t)$ and the optimal investment threshold Λ_m^* . It is well known that the optimal investment threshold Λ_m^* is higher than the value of the Poisson intensity $\bar{\Lambda}$ at which the NPV = 0, see e.g. Dixit and Pindyck (1994), and that at the optimal investment threshold, the NPV of the project is positive².

When Λ_t is higher than $\bar{\Lambda}$, the advice from the NPV rule is to invest in the project and the loss due to using the NPV rule is the difference between the option value and the NPV, i.e. $F(\Lambda_t) - V(\Lambda_t)$. In contrast, when Λ_t is lower than $\bar{\Lambda}$, the project has a negative NPV and is, therefore, not invested under the NPV rule. However, in this case, the loss due to the use of the NPV rule is not zero since the project will be suboptimally invested whenever Λ_t reaches $\bar{\Lambda}$ at a future time.

The expected loss at a value $\Lambda_t < \overline{\Lambda}$ due to the use of the NPV rule is the value of the option at $\overline{\Lambda}$ discounted by the time τ that the Poisson intensity takes to reach $\overline{\Lambda}$ from its current level Λ_t :

$$F(\bar{\Lambda})E[e^{-r\tau}].$$
(2.18)

As demonstrated by Dixit and Pindyck (1994, p.315), $E[e^{-r\tau}] = (\Lambda_t/\bar{\Lambda})^{\alpha}$, where α is given in (2.9). Since $F(\bar{\Lambda}) = B_m \bar{\Lambda}^{\alpha}$, the loss is then $B_m \Lambda_t^{\alpha}$, which is the value of the option to invest. The loss due to the use of the NPV rule is therefore equal to the value of the option for negative NPV projects and equal to the difference between the value of the option and the NPV for positive NPV projects.

3. Case Study Analysis

In this section, we apply the proposed model to a case study of bushfire risk management in a local government area (Ku-ring-gai) in Southeastern Australia. Ku-ring-gai is an urban area with residential properties surrounded by three national parks. It has 89 kilometres of urban and bushland interface and ranks third in bushfire vulnerability among the 61 local government areas in the Greater Sydney region (Chen, 2005).³ The community in Ku-ring-gai recognizes bushfires as the most concerning risk under climate change, followed by storms, water supply security and heat stress mortality risk (Ku-

²With Λ_m^* given in (2.10), the NPV of the project when $\Lambda_t = \Lambda_m^*$ is $(I_m + C_m/r)/(\alpha - 1)$ which is positive since $\alpha > 1$. For the proof that $\alpha > 1$, see Dixit and Pindyck (1994, p.143).

 $^{^{3}}$ Bushfire vulnerability is defined as the number of addresses within 130 meters of bushland.

ring-gai Council, 2010).

A number of options have been identified by Ku-ring-gai Council to reduce the risks from bushfires. These include, among others, building new fire-trails, constructing new rural fire-stations and rezoning land, see Ku-ring-gai Council (2010). Fire trails allow for controlled hazard reduction burning, break wild fire transition and potentially allow more time for fire fighters to respond to bushfires. Constructing more rural fire stations provides volunteer rural fire fighters with effective fire suppression equipment and will reduce the response time. Thus, such an investment project may also significantly reduce the risk of a fire to become more severe. In the following, we will consider a situation, where two alternative adaptation investment projects are being examined: a project that involves the construction of additional fire trails and a project that requires investment in an additional rural fire station. We will use this case study to illustrate the proposed framework and to provide economic insights on the value of investment flexibility.

3.1. Parameter Calibration

Bushfire risk in Australia is highly impacted by climate variables, especially temperature and windspeed, and is predicted to increase significantly in future years due to the impact of climatic change (Lucas, 2010). Reliable forecasts about how the climate will change, are, however, difficult to obtain, since the change depends on, e.g., carbon emissions with a significant time lag, while predictions based on historical climate observations may be unsatisfactory (Wei et al., 2015; Matsumoto and Andriosopoulos, 2016). Studies that forecast climate change usually utilize climate models that simulate the interactions among important drivers of climate, including atmosphere, oceans, land surface and ice (Solomon, 2007). In this paper, we rely on climate change predictions to calibrate the Poisson intensity process Λ_t . Our method of using forecasts from climate change studies to calibrate the risk process is similar to the method used by Mills et al. (2014) who consider the problem of optimal adaptation investment to reduce the impact of sea level rise in a two period framework.

We consider a forecast for the growth rate of the intensity for extreme bushfires, $\lambda_T = \ln \Lambda_T$, provided at the current point in time t by a given climate model using a given

emission scenario, as a possible value that λ_T can take at a future point in time T. Using a range of climate models with several emission scenarios will provide a number of forecasts of λ_T . These forecasts can then be used to estimate the distribution of λ_T given the current information at time t. Since Λ_T follows a GBM, see Equation (2.3), λ_T is normally distributed with mean $(\mu - 0.5\sigma^2)(T - t)$ and variance $\sigma^2(T - t)$. Using the estimated distribution of λ_T , we can then determine the parameter estimates for μ and σ .

We use 20 forecasts of the frequency of extreme fire weather events in Southeastern Australia as provided by Hasson et al. (2009). These forecasts are generated from 10 general circulation models using a low (B1) and a high (A2) GHG emission scenario. We assume that each model uses different radiative forcing parameter values that are equally probable and that the two emission scenarios are also equally likely. The 20 forecasts then form an empirical distribution for Λ_T and the annual growth rates for the frequency of extreme bushfire events can be calculated from these predictions. Based on the provided forecasts, our estimate for the average annual growth rate for the period 2010 - 2100 is 1.59% with a standard deviation of 1.54%. Fitting a normal distribution to the predicted annual growth rates then yields an estimated distribution with a mean of 1.59% and a standard deviation of 1.50%. The estimated distribution for λ_T is illustrated in Figure 2.

The current value of the Poisson intensity Λ_0 can be estimated using the PerilAus database that records bushfire events since 1906. It is observed that until 2016, there were 3 bushfire events with significant damage to houses and infrastructure in the area, so the current estimate for the Poisson intensity is $\Lambda(0) = 0.027$.

To determine the discount rate, we use the estimation results provided by Truong and Trück (2016a) who estimate the stochastic interest rate model proposed by Cox et al. (1985), and use the expected discount factor given by the stochastic interest rate model to find the certainty equivalent discount rate. The estimated certainty equivalent discount rate is found to converge quickly to the long run level (4.5%), and for simplicity, we assume that the discount rate is constant at 4.5%.

To estimate the loss severity distribution, we assume that houses are either completely



Figure 2: Estimated distribution for the annual growth rate of bushfire intensity λ_T for the year 2100. We use 20 alternative forecasts generated from 10 general circulation models using a low (B1) and a high (A2) GHG emission scenarios as in Hasson et al. (2009), yielding estimates of $\mu = 1.59$ and $\sigma = 1.50$ for the distribution of λ_T .

destroyed by bushfires or will survive unscathed based on the empirical observation by Crompton et al. (2010). The loss severity is then a product of the number of damaged houses and the reconstruction cost per house. The reconstruction cost per house is estimated by subtracting the average land value estimated by the NSW Valuer General (DOL, Department of Land (2009)) from the average net-of-realtor-commission property sales price in the region provided by Hatzvi and Otto (2008)⁴. The reconstruction cost per house is estimated as \$405,000.

The number of damaged houses in a bushfire event is estimated based on the information provided by a local expert from the bushfire brigade. The expert suggests that for a severe bushfire, the average number of houses being damaged is 30, which implies an expected loss without growth of loss exposure of \$12.15 million.

To estimate the investment costs, loss mitigation effectiveness and project life, we use the expert elicitation method that has been used in many previous climate adaptation

 $^{^4\}mathrm{Note}$ that we assume an additional 2.5% real tor commission for property sales.

Table 1: Information on estimated and assumed parameter values, including the current intensity of bushfires $\lambda(0)$, the expected intensity growth rate μ , the volatility of intensity growth rate σ , expected loss $E(X_0)$ and the estimated growth rate of the cost of reconstruction γ . For each type of projects (fire trails vs rural fire station), the table also provides information about the assumed impacts on risk mitigation of the project k, the lifetime of each project M, the investment cost per project I_M , project maintenance costs C and the applied discount rate r.

Parameters	Value
Current Poisson intensity $(\lambda(0))$	0.027
Expected Rate of Poisson intensity growth (μ)	1.59~%
Volatility of Poisson intensity growth(σ)	1.50~%
Expected loss severity $(E(X_0))$	12.15 M
Growth rate of reconstruction cost (γ)	1%
Risk mitigation by the fire station (k_1)	18%
Risk mitigation by the fire trails (k_2)	20%
Lifetime of the fire station (M_1)	40 years
Lifetime of the fire trails (M_2)	50 years
Investment cost per project for rural fire station (I_M^1)	0.75 million
Investment cost per project for fire trails (I_M^2)	\$1.5 million
Maintenance cost of rural fire station (C_1)	\$70,000
Maintenance cost of fire trails (C_2)	\$50,000
Discount rate (r)	4.5%

studies, see e.g. Baker and Solak (2011); Mathew et al. (2012), to overcome the problem of data scarcity. The expert specifies that additional fire trails are expected to reduce the frequency of house damaging bushfire events by 20%, while investment in an additional fire station is expected to reduce the frequency of house damaging bushfire events by 18%. We assume that the risk reduction for the two adaptation investments is independent due to the different nature of the investments. While fire trails will lead to a reduced risk as a result of an increased and more effective hazard reduction burning program, a new fire station will allow a better protection of properties and infrastructure near a park.

The investment cost of an infinite lifetime project can be calculated from the investment cost I_M for a finite lifetime project estimated by the expert (Table 1) by firstly converting I_M into an annuity flow, A:

$$A = I_M \frac{1 - (1 + r)^{-1}}{1 - (1 + r)^{-(M+1)}}.$$

The annuity A is then used to calculate the investment cost of an infinite life project:

$$I = A(1+r)/r.$$
 (3.1)

At the discount rate of 4.5%, the present value of building a fire station every 40 years for \$0.75 million each is \$0.90 million while the present value of building bushfire trails every 50 years for \$1.5 million, is \$1.68 million.

3.2. Baseline Case

In a first step we discuss the obtained results for the estimated models and calibrated parameters in the previous section. A summary of the results for this baseline case is provided in Table 2. Given the estimated parameters, the NPV of immediate adaptation investment in the fire station is \$641,563, while the NPV for investment in the fire trails is \$649,824. Should the NPV rule be used, the investment into the fire trails would be preferred to the fire station since it yields a higher NPV under immediate investment. After the investment into the fire trails, bushfire risk is reduced by 20% and the NPV of the fire station is reduced to \$22,600, since there remains less risk for the fire station to mitigate. However, since the NPV of the fire station is positive, it would still be invested according to the NPV rule. Thus, using the NPV rule to guide investment, both the fire trails and the fire station would be invested immediately with an overall NPV for investing in both projects at time t = 0 of \$672,424.

Table 2: Results for baseline case for different investment settings. Investment can be conducted individually into one of the projects, i.e. a new fire station or fire trails according to the NPV rule at t = 0 or according to the decision rule provided by the real options model. Alternatively, both investments can be undertaken at t = 0 ($F_1 + F_2$) or sequential investment (either $F_1 + F_{12}$ or $F_2 + F_{21}$) can be conducted, taking into account the optimal sequential timing for each investment.

	Investment model		
Investment value	NPV rule	Real option	
Individual Investment			
F_1 (Fire station)	\$641,563	\$759,445	
F_2 (Fire trails)	\$649,824	\$809,978	
Combined/Sequential Investment			
$F_1 + F_2$	\$672,424	-	
$F_1 + F_{12}$	-	\$1,224,939	
$F_2 + F_{21}$	-	\$1,217,363	

The NPV profiles of the two projects are depicted in Figure 3. The fire station dominates when Λ_t is lower than 0.026, otherwise the fire trail investment dominates. In addition, since condition (2.12) is satisfied, the option to invest in the fire trails is higher than the option to invest in the fire station for all Λ_t (despite the dominance of the NPV profile of the fire station for $\Lambda_t < 0.026$). If the decision maker has to choose between investment into fire trails or the fire station, the fire trails project would be selected. The value of the option to invest in alternative projects is then equal to the value of the option to invest in the fire trails (\$809,978). It is optimal to invest in the fire trails when the Poisson intensity is equal to or higher than $\Lambda_2^* = 0.034$. Using the NPV rule instead of the real options framework to determine investment would result in a loss of 19.77% (\$809,978-\$649,824=\$160,154) of the option value.

In contrast, if the decision maker can invest in the fire trails as well as the fire station in a sequential order, he/she should invest in the fire station first when the Poisson intensity reaches 0.033 and invest in the fire trails when the Poisson intensity is at or above 0.042. At the current value of the Poisson intensity, the value of the option to invest in the fire station first and the fire trails later is \$1,224,939, while the value of the option to invest in the sequence of fire trails first and fire station later is \$1,217,363. As expected, the value of the sequential investment option is less than the sum of individual investment



Figure 3: NPV profiles of investment into a fire station (solid) and into a fire trail investment projects for different values of Λ_t . The fire station dominates when Λ_t is lower than 0.026, otherwise the fire trail investment dominates.

options (\$1,569,423), due to the fact that after a project has been invested, there remains less risk to be mitigated. The consideration of sequential investment increases the option to invest by 51.23% (\$414,960) and increases the loss due to the use of the NPV rule to 45.11% (\$1,224,939-\$672,424=\$552,515) of the option value. A summary of the results for different investment strategies, i.e. individual, combined and sequential investment according to the NPV rule and taking into account the optimal timing and sequencing of the projects, is provided in Table 2.

3.3. Impact of Uncertainty

The impact of uncertainty on the option values and on the loss due to the usage of a simple NPV rule is examined by increasing the uncertainty parameter σ by 10% (from 1.50% to 1.65%). With an increase in uncertainty, the option value of the fire station increases by 0.15% (to \$760,600) while the option value of the fire trails increases by a higher proportion of 0.17% (to \$811,342). This is due to the higher investment cost of the fire trails compared to that of the fire station.

Even with an increase in uncertainty, the fire trails project is still preferred when only one project is selected for investment. For the sequential investment, the order of investment also remains unchanged. The value of the option to invest in the fire trails is increased by 0.17% and since the NPV of the project is not affected by the increase in uncertainty

(under the risk neutrality assumption), the loss in the option value due to the use of the NPV rule is increased by 0.85% (to \$161,518). The value of the option to invest in the optimal sequence of investment projects is increased by 0.21% and since the NPVs of the projects are not affected by the change in uncertainty, the loss in the option value of sequential investment due to the use of the NPV rule is increased by 0.45% (to \$577,717).

The impact of uncertainty is examined further by allowing σ to vary over a range from 0 to 10%. As shown in Figure 4, the value of the option to invest in alternative investments and the value of the option for sequential investment are both increasing in uncertainty, with the latter increasing at a higher rate. As a consequence, the loss due to immediate investment based on the NPV rule is higher when uncertainty is higher and when sequential investment is feasible.



Figure 4: Impact of different values of uncertainty parameter ($0 < \sigma < 0.1$) on option values and the loss due to using a NPV rule. We plot the value of the option to invest in alternative projects (solid), the value of the sequential investment option (dashed), the loss in the option value of alternative projects (dotted), and the loss in the option value of sequential investment due to the use of NPV rule (dotdash).

3.4. Impact of Climate Change

We examine the impact of a more serious climate change scenario by increasing the growth rate μ by 10% to 1.75%. In this scenario, the NPVs of the projects are significantly increased: the NPV of the fire station is increased by 43.91% (to \$923,295) and the NPV of the fire trails is increased by 48.17% (to \$962,860). The option values increase by a lesser extent, with the option to invest in the fire station increasing by 32.37% (to \$1,005,272) and that of the fire trails increasing by 33.12% (to \$1,078,228).

In the scenario of more serious climate change, the option to invest in the fire trails remains larger than the option to invest in the fire station. The optimal investment strategy for alternative investments is therefore to wait and invest in the fire trails. For sequential investment, investing in the fire station first and the fire trails later also remains to be the optimal strategy. The loss in the value of the alternative investment option is reduced by 27.96% to \$115,367 while the loss in the value of the sequential investment option due to the NPV rule usage is reduced by 19.42% to \$445,191. This is a result of the assumption of a more serious climate change scenario, that favours earlier adaptation investment and therefore reduces the difference between the investment decision given by the NPV rule and that given by the real options model.

The impact of a more serious climate change scenario is further explored by allowing μ to vary over a range from 0 to 2% (Figure 5). Under a more serious climate change scenario, adaptation projects are more beneficial and the investment options become more valuable. Regarding the loss due to the NPV rule, it depends not only on the option value but also on whether or not the projects are invested under the NPV rule, as explained in section 2.4. When μ is lower than 1.1%, the NPVs of the two projects are negative such that no projects are invested and losses are equal to the values of the options. When μ is higher than 1.1%, the NPVs of the two projects are positive with the NPV of the fire station higher than the NPV of the fire trails. Using the NPV rule, the fire station would be invested while additional investment in the fire trails would be justified only when μ is higher than 1.58% (note that the NPV of the fire trails decreases when the fire station is invested).

The loss in the value of the option to invest in alternative investments is equal to the loss in the value of the option to invest in the fire trails, since this option value is higher than the option value of the fire station. When μ is lower than 1.1%, the NPV of the firetrails is negative. Increases in μ will increase the value of the option and therefore increase the loss. As μ increases beyond 1.1%, further increases in μ lowers the optimal investment threshold, as evident in (2.10), and reduces the difference in the optimal investment decision and the investment decision given by the NPV rule. The loss is reduced as a result.

The loss in the value of the option to invest in sequential projects is equal to the value of that option when μ is lower than 1.1%. When μ is higher than 1.1% and lower than 1.58%, the loss is equal to the sum of the option value for the investment in the fire trails after the fire station has been invested and the difference between the option value of the fire station and its NPV. The increase in the loss, when μ increases for this range of values, indicates that the loss in the option value of the fire trails dominates the loss in the option value of the fire station. When μ is above 1.58%, a further increases in μ reduces the difference between investments under the NPV rule and that under the real options model. The loss is therefore reduced.



Figure 5: Impact of different values for the growth rate of the Poisson intensity ($0 < \mu < 0.02$) as a proxy for different climate change scenarios on option values and the loss due to using the NPV rule. We plot the value of the option to invest in alternative projects (solid), the value of the sequential investment option (dashed), the loss in the option value of alternative projects(dotted), and the loss in the option value of sequential investment (dotdashed) due to the use of the NPV rule.

3.5. Impact of the Discount Rate

The impact of the applied discount rate is examined by increasing the discount rate by 10% (to 4.95%). The increase in the discount rate results in a sharp decrease in the NPVs of the projects: the NPV of the fire station is decreased by 65.70% (to \$220,038)

and the NPV of the fire trails decreases by 79.56% (to \$132,830). This high sensitivity of the projects' NPV to the applied discount rate is due to the projects having larger benefits in the far future when catastrophic risks are higher. The option values are also decreased, although by a lesser extent. The value of the option to invest in the fire station is decreased by 43.85% (to \$426,413) while the value of the option to invest in the fire trails is decreased by 46.73% (to \$431,458).

Since the option to invest in the fire trails still has a higher value than the option to invest in the fire station, when only one project can be invested, the fire trails project is still selected. For sequential investment, investing in the fire station first and the fire trails later also remains as the optimal sequence. The loss in the value of the alternative investment option, which is the difference between the option to invest in the fire trails and its NPV, is increased by 86.46% (to \$298,628) due largely to the reduction in the NPV of the fire trail. In contrast, the loss in the value of the sequential investment option is decreased by 20.14% (to \$441,244). This is because under the higher discount rate, the NPV of additional investment in the fire trails changes from positive to negative and the loss in the value of the option to further invest in the fire trails equals to the option value, which is less sensitive to changes in discount rate than the NPV.

The impact of the applied discount rate is further investigated by allowing the discount rate to vary from 4% to 5% (Figure 6). In general, a higher discount rate reduces the NPV of investment projects and increases the loss in the values of the investment options.

3.6. Impact of Investment Cost

The impact of investment costs on option values is examined by increasing the investment costs by 10%. The increase in investment cost reduces the NPV of the fire station by 13.99% to \$551,793, and reduces the NPV of the fire trails by 25.82% to \$482,050. The value of investment options are also reduced, although by a smaller amount. The value of the option to invest in the fire station is reduced by 6.23% (to \$712,098) while the value of the option to invest in the fire trails is reduced by 9.93 % (to \$729,507).

At the higher investment costs, the value of the investment option of the fire trails is



Figure 6: Impact of different values of the discount rate (4% < r < 5%) on option values and the loss due to using an NPV rule. We plot the value of the option to invest in alternative projects (solid), the value of the sequential investment option (dashed), the loss in the option value of alternative projects(dotted), and the loss in the option value of sequential investment (dotdashed) due to the use of NPV rule.

still higher than that of the fire station and the construction of fire trails remains the preferred project for alternative investments. The loss in the value of the alternative investment option due to the NPV rule is increased by 54.51% to \$247,457. For sequential investment, investing in the fire station first is still optimal. The loss in the value of the sequential investment option increases by 4.89% to \$579,552. The smaller impact of the higher investment costs on the loss in sequential investment option value is largely due to the NPV of additional investment in the fire trails becoming negative at the higher investment costs. The loss in the option value of further investment in the fire trails is therefore equal to the value of that option, rather than the difference between the option value and the NPV than on the option value, the loss in the value of the sequential investment option increases by a smaller amount.

We further investigate the impact of investment cost by allowing the change in investment costs to vary from -40% to 40% (Figure 7). For investment projects with a positive NPV, the loss due to applying the NPV rule is higher for investment projects with higher investment costs. For alternative investments, when the increase in investment costs passes 17%, the investment cost of the fire trails becomes too large relative to the

reduction in bushfire risk delivered by the project and it is preferred to wait to invest in the fire station, rather than the fire trails. The switch in the preferred investment project causes a slight jump in the loss in the value of alternative investment. Similarly, when the increase in investment costs passes 38%, the optimal sequence of investment changes, causing a jump in the loss in the option value of sequential investment.



Figure 7: Impact of reduction and increase in the investment costs $(-40\% < \Delta(I_M) < 40\%, j = \{1, 2\})$ on option values and the loss due to using NPV rule. We plot the value of the option to invest in alternative projects (solid), the value of the sequential investment option(dashed), the loss in the option value of alternative projects(dot), and the loss in the option value of sequential investment (dotdashed) due to the use of NPV rule.

3.7. Summary of Sensitivity Analysis

The sensitivities of the NPV, the option values of investment and the loss incurred by using the NPV rule, to a 10% increase in the considered variables are summarized in Table 3. The conducted sensitivity analysis illustrates that the results are relatively insensitive to the uncertainty parameter, while they are quite sensitive to the assumptions about the impact of climatic change, and to the applied discount rate. This implies that for the purpose of investment analysis under uncertainty of climate change, it is important to include as many climate change predictions as possible. This is true in particular, if a higher number of predictions increases the accuracy of the expected forecasts. The investment cost also has a large impact on the loss due to the application of the NPV rule, suggesting that the real options framework is more important for high sunk cost projects. Table 3: Sensitivity of the results for NPV and option value of the investment to a 10% increase in the value of considered key variables. The uncertainty parameter $\sigma = 1.50\%$, the discount rate (4.5%) and investment costs (\$0.75M for fire station and \$1.5M for fire trail) are increased by 10%. For the climate change variable, we assume that the expected growth rate of Poisson intensity $\mu = 1.59\%$ is increased by 10% to 1.75%

10% Increase	Δ NPV	Δ Option Value	Δ Loss NPV rule vs.
			optimal investment
Alternative Investment			
Uncertainty	0%	0.17%	0.85%
Climate Change	48.17%	33.12%	-27.96%
Discount rate	-79.56%	-46.73%	86.46%
Investment cost	-25.82%	-9.93%	54.51%
Sequential Investment			
Uncertainty	0%	0.21%	0.45%
Climate Change	80.53%	35.18%	-19.42%
Discount rate	-122.01%	-46.01%	-20.14%
Investment cost	-38.30%	-7.64%	4.89%

4. Conclusion

In this paper, we provide a new modelling framework that allows to analyze and select optimal investment in catastrophic risk reduction projects under the uncertainty of climate change. An valuable feature of our model is that it allows to incorporate the uncertainty about climate change predictions into the analysis of investment decisions. The framework allows to analyze alternative investments where the decision maker can select only one project among many, possibly due to budget constraints. Importantly, it further allows to select optimal sequential investments where the decision maker is free to choose any investment projects that improve the social welfare.

We illustrate the application of the framework to a case study of bushfire risk management in a local government area in Australia. The results suggest that the value of investment flexibility under uncertainty is large relative to the NPV of the considered projects. Using the NPV rule will result in suboptimal outcomes, in comparison to the application of a real options framework. The loss is higher for the case where the decision maker is not constrained to select one investment project only, but can invest in several projects at the same time. These results clearly emphasize the importance of applying real options analysis. An interesting finding from the empirical application is that if the decision maker is constrained by their financial budget and can invest in only one project, it is optimal to select the largest project in order to obtain the largest reduction in catastrophic risk. On the other hand, if the decision maker is free to select investment projects to maximize the long term social welfare, it is optimal to select a low sunk cost project first to preserve the investment flexibility and only invest in a high sunk cost project when the climate change impact turns out to be quite serious. Compared with the case of a single project investment, investment starts earlier when a sequence of projects is considered. Furthermore, the optimal investment strategy for sequential investment is consistent with results obtained by a more qualitative analysis in previous studies, see e.g. Hallegatte (2009), that suggest to invest in low sunk cost projects first to preserve flexibility. Sequential investment analysis is, therefore, essential for climate change adaptation projects. These results also suggest that in order to enable effective adaptation, financial assistance from higher government levels to local governments may be necessary, so that local governments are less constrained financially and are able to preserve investment flexibility. Our findings illustrate that in particular for managing sequential investments, local governments should base their investment decisions on a real options model instead of a simple NPV rule.

Optimal investment decisions, including which project to select for alternative investment as well as the order of the invested projects in a sequential investment framework, are robust to changes in the considered key parameters. Further, the NPVs and option values of the projects are relatively insensitive to the uncertainty parameter, while they are quite sensitive to the impact of climatic change and the applied discount rate. Therefore, for the purpose of investment analysis under uncertainty about climatic change, it is important to include as many predictions about climate change impact as possible, in particular if a higher number of predictions increases the accuracy and robustness of the expected forecasts. Initial investment costs for adaptation projects also have a large impact on the loss induced by the application of a NPV rule, which implies that the application of a real options framework is more important for high sunk cost projects. Note that in our empirical analysis, we use a univariate approach to estimate the Poisson intensity of bushfire events. With this approach, the Poisson intensity only increases when a new event occurs, although the stochastic variation of climate variables will influence the pattern of extreme climate events. A better approach is probably to use a generalized linear model to relate catastrophic events to climate variables. Once the statistical relation is established, the Poisson intensity can be inferred from observed climate variables. This more statistically rigorous approach is beyond the scope of the current paper and is left to future research.

References

- Baker, E., Solak, S., 2011. Climate change and optimal energy technology R&D policy. European Journal of Operational Research 213 (2), 442–454.
- Baranzini, A., Chesney, M., Morisset, J., 2003. The impact of possible climate catastrophes on global warming policy. Energy Policy 31 (8), 691–701.
- Bouwer, L. M., Bubeck, P., Aerts, J. C., 2010. Changes in future flood risk due to climate and development in a dutch polder area. Global Environmental Change 20 (3), 463–471.
- Brouwer, R., van Ek, R., 2004. Integrated ecological, economic and social impact assessment of alternative flood control policies in the Netherlands. Ecological Economics 50 (1-2), 1–21.
- Chen, K., 2005. Counting bushfire-prone addresses in the greater Sydney region. In: Proceedings of the Symposium on Planning for Natural Hazards: how can we mitigate the impacts? University of Wollongong.
- Cox, J. C., Ingersoll, J. E., Ross, S. A., 1985. A theory of the term structure of interest rates. Econometrica 53 (2), 385–407.
- Crompton, R., McAneney, J., Leigh, R., 2006. Natural disaster losses and climate change: an Australian perspective. In: Proceedings of the Workshop on Climate Change and Disaster Losses-Understanding and Attributing Trends and Projections. pp. 25–26.
- Crompton, R. P., McAneney, K. J., 2008. Normalised Australian insured losses from meteorological hazards: 1967–2006. Environmental Science & Policy 11 (5), 371–378.
- Crompton, R. P., McAneney, K. J., Chen, K., Pielke Jr, R. A., Haynes, K., 2010. Influence of location, population, and climate on building damage and fatalities due to Australian bushfire: 1925-2009. Weather, Climate, and Society 2 (4), 300–310.
- Décamps, J.-P., Mariotti, T., Villeneuve, S., 2006. Irreversible investment in alternative projects. Economic Theory 28 (2), 425–448.
- Dixit, A. K., Pindyck, R. S., 1994. Investment under uncertainty. Princeton University Press, Princeton, New Jersey.
- DOL, Department of Land, 29 October 2010 2009. Land values issued for Ku-ring-gai.
- Fisher, A. C., 2000. Investment under uncertainty and option value in environmental economics. Resource and Energy Economics 22 (3), 197–204.
- Fisher, A. C., Rubio, S. J., 1997. Adjusting to climate change: Implications of increased variability and asymmetric adjustment costs for investment in water reserves. Journal of Environmental Economics and Management 34 (3), 207–227.
- Gersonius, B., Ashley, R., Pathirana, A., Zevenbergen, C., 2013. Climate change uncertainty: building flexibility into water and flood risk infrastructure. Climatic Change 116 (2), 411–423.
- Gollier, C., Treich, N., 2003. Decision-making under scientific uncertainty: The economics of the precautionary principle. Journal of Risk and Uncertainty 27 (1), 77–103.
- Haasnoot, M., Kwakkel, J. H., Walker, W. E., ter Maat, J., 2013. Dynamic adaptive policy pathways: a method for crafting robust decisions for a deeply uncertain world. Global Environmental Change

23(2), 485-498.

- Haasnoot, M., Middelkoop, H., Offermans, A., Van Beek, E., van Deursen, W. P., 2012. Exploring pathways for sustainable water management in river deltas in a changing environment. Climatic Change 115 (3-4), 795–819.
- Hallegatte, S., 2009. Strategies to adapt to an uncertain climate change. Global Environmental Change 19 (2), 240–247.
- Härdle, W. K., Cabrera, B. L., 2010. Calibrating cat bonds for Mexican earthquakes. Journal of Risk and Insurance 77 (3), 625–650.
- Hasson, A. E. A., Mills, G. A., Timbal, B., Walsh, K., 2009. Assessing the impact of climate change on extreme fire weather events over southeastern Australia. Climate Research 39 (2), 159–172.
- Hatzvi, E., Otto, G., 2008. Prices, rents and rational speculative bubbles in the Sydney housing market. Economic Record 84 (267), 405–420.
- Kirshen, P., Knee, K., Ruth, M., 2008. Climate change and coastal flooding in Metro Boston: impacts and adaptation strategies. Climatic Change 90 (4), 453–473.
- Klugman, S. A., Panjer, H. H., Willmot, G. E., 2008. Loss models : from data to decisions, 3rd Edition. Wiley series in probability and statistics. John Wiley & Sons, Hoboken, N.J.
- Ku-ring-gai Council, 2010. Climate change adaptation strategy. Tech. rep., Ku-ring-gai Council.
- Kuijken, W., 2010. The Delta Programme in the Netherlands: the Delta Works of the Future. Speech at the 'Deltas in Times of Climate Change' Conference, Rotterdam, the Netherlands, Wednesday, 29 September 2010.
- Kwakkel, J., Haasnoot, M., 2012. Computer assisted dynamic adaptive policy design for sustainable water management in river deltas in a changing environment. International Environmental Modelling and Software Society (iEMSs).
- Lempert, R. J., Groves, D. G., 2010. Identifying and evaluating robust adaptive policy responses to climate change for water management agencies in the American west. Technological Forecasting and Social Change 77 (6), 960–974.
- Lin, S.-K., Chang, C.-C., Powers, M. R., 2009. The valuation of contingent capital with catastrophe risks. Insurance: Mathematics and Economics 45 (1), 65–73.
- Lucas, C., 2010. On developing a historical fire weather data-set for Australia. Australian Meteorological and Oceanographic Journal 60 (1), 1.
- Mathew, S., Trück, Henderson-Sellers, A., 2012. Kochi, India case study of climate adaptation to floods: Ranking local government investment options. Global Environmental Change 22 (1), 308 – 319.
- Matsumoto, K., Andriosopoulos, K., 2016. Energy security in East Asia under climate mitigation scenarios in the 21st century. Omega 59, 60–71.
- Michael, J. A., 2007. Episodic flooding and the cost of sea-level rise. Ecological Economics 63 (1), 149–159.
- Mills, M., Nicol, S., Wells, J. A., Lahoz-Monfort, J. J., Wintle, B., Bode, M., Wardrop, M., Walshe, T., Probert, W. J., Runge, M. C., et al., 2014. Minimizing the cost of keeping options open for

conservation in a changing climate. Conservation biology 28 (3), 646–653.

- Pindyck, R. S., 2002. Optimal timing problems in environmental economics. Journal of Economic Dynamics & Control 26 (9-10), 1677–1697.
- Shevchenko, P., Wüthrich, M., 2006. The structural modeling of operational risk via Bayesian inference: combining loss data with expert opinions. Journal of Operational Risk 1 (3), 3–26.
- Solomon, S., 2007. Climate change 2007-the physical science basis: Working group I contribution to the fourth assessment report of the IPCC. Vol. 4. Cambridge University Press.
- Truong, C., Trück, S., 2016a. Its not now or never: Implications of investment timing and risk aversion on climate adaptation to extreme events. European Journal of Operational Research 253 (3), 856–868.
- Truong, C., Trück, S., 2016b. The principal-agent problem in coastal development: Response to Mills et al. Conservation Biology, forthcoming.
- Tsvetanov, T. G., Shah, F. A., 2013. The economic value of delaying adaptation to sea-level rise: An application to coastal properties in Connecticut. Climatic Change 121 (2), 177–193.
- Van Aalst, M. K., 2006. The impacts of climate change on the risk of natural disasters. Disasters 30 (1), 5–18.
- Wei, Y.-M., Mi, Z.-F., Huang, Z., 2015. Climate policy modeling: An online SCI-E and SSCI based literature review. Omega 57, 70–84.
- West, J. J., Small, M. J., Dowlatabadi, H., 2001. Storms, investor decisions, and the economic impacts of sea level rise. Climatic Change 48 (2-3), 317–342.
- Zhu, T. J., Lund, J. R., Jenkins, M. W., Marques, G. F., Ritzema, R. S., 2007. Climate change, urbanization, and optimal long-term floodplain protection. Water Resources Research 43 (6).