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Retirement Wealth Outcomes for Superannuation Portfolios - A Risk-Adjusted Analysis

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Abstract

We examine factors impacting on the distribution of retirement wealth outcomes for superannuation portfolios. We focus on the last 10 years prior to retirement as they play an important role in determining the final wealth outcome for an investor. We evaluate the performance of different investment strategies for a superannuation portfolio under different scenarios of market conditions, the initial accumulated value of the portfolio, salary and contribution levels as well as sequencing risk. We apply parametric and non-parametric techniques to evaluate different strategies by examining wealth outcomes and risk-adjusted performance measures. Our results point towards a superior performance of strategies that invest a higher share in growth assets even when risk-adjusted performance measures are considered. We find that recent data suggests more conservative expectations for terminal wealth. Accumulated wealth 10 years prior to retirement has a linear effect on terminal wealth, this effect is significant with respect to other factors such as regimes of high or low volatility. Another factor with high impact is the presence of a crisis, particularly if it occurs in the beginning. We also find that if contributors with lower to median income slightly increase their contribution level, they significantly increase their chances of obtaining a terminal wealth equivalent or higher to the required amount for a comfortably lifestyle according to ASFA.

Keywords: Superannuation Funds, Lifecycle Investment, Risk Management, Copulas, Nonparametric Estimation

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1. Introduction

In recent years, dynamic asset allocation and lifecycle strategies have become increasingly popular for defined contribution plans in superannuation funds. Such strategies have the advantage of adjusting the portfolio composition towards the age of retirement. They typically allow a superannuation member to realign the underlying asset allocation from growth stocks to more conservative assets and cash as part of retirement planning, once they approach the retirement age. As pointed out by Chant et al (2014), a key motivation of a lifecycle approach is to reduce sequencing risk, i.e. to reduce the probability for members to suffer a large loss in fund value when it matters most near retirement. Since older members most likely rely more heavily on their superannuation fund, their propensity for bearing risk in that fund will be reduced.

Traditionally the Australian context of MySuper products has been dominated by Target Risk Funds (TRFs) for which the weight given to different assets is always constant. In recent times Target Date Funds (TDFs) have replaced TRFs.TDFs strategies have the advantage of adjusting the portfolio composition towards the age of retirement and typically switch from growth to more defensive assets. The adjustment of the portfolio done by TDFs is known as "set it and forget it". In recent times other dynamic lifecycle strategies have appeared that also offer the flexibility to take into account contribution levels, the accumulated value of the portfolio and current market conditions, see Basu and Drew (2009) and Basu et al. (2009).

There is an ongoing debate in the literature regarding the optimal nature of Lifecycle strategies. Many authors advocate for the standard approach in which there is a switch from growth to conservative assets towards the age of retirement. They argue that is too risky to invest in growth assets in the final years before retirement. However, it has also been argued that by doing this investors may miss out on the opportunity to maximise their wealth when the balance is already at a sufficiently high level (Basu and Drew, 2009).

In this study, we examine the impact of a number of key factors on the distribution of retirement wealth outcomes for superannuation portfolios by focusing on the last 10 years prior to retirement. We examine the performance of different investment strategies, in particular two target risk funds (TRFs) strategies with (i) a relatively high, and, (ii) a significantly lower share in growth assets in comparison to a lifecycle strategy that linearly reduces the share of equity investments in the portfolio. We evaluate the performance of these investment strategies for a superannuation portfolio under different scenarios of prevailing market conditions, the initial accumulated value of the portfolio, salary and contribution levels as well as sequencing risk. In our analysis we do not focus only on the expected outcomes for the strategies, but also consider risk-adjusted measures, for example probabilities of exceeding defined threshold levels such as e.g. the lump sum required to support a comfortable lifestyle as suggested by the Association of Superannuation Funds of Australia (ASFA, 2014).

In our analysis we apply both parametric and non-parametric techniques to model and simulate the returns of different asset classes through time. We also analyse the role of several factors in determining the final wealth at retirement during the last 10 years of contributions. Hereby, we examine a wide range of scenarios of market conditions, accumulated values, salaries and salary growth rates as well as interest rates and contribution levels. We then evaluate retirement wealth outcomes across a number of criteria, including the expected portfolio value at retirement, the distribution of simulated retirement wealth ratios (RWR) and the probability of exceeding a number of different threshold values at retirement. The inclusion of risk related performance measures is fundamental in the analysis of strategies for superannuation funds, since near retirement superannuation members might become more risk averse and their willingness to take risks with regards to the final wealth outcome will be reduced.

Our results suggest that even during the last 10 years before retirement, a strategy that predominantly invests in growth assets outperforms the more conservative target risk fund strategy. Interestingly, and contrary to common belief, our results also seem to indicate that the more aggressive TRF strategy tends to outperform a lifecycle investment strategy that switches from growth to defensive assets over the considered 10 year period. The difference in performance of the strategies becomes clear when calculating the probability of exceeding different RWR thresholds. While the conservative TRF and the lifecycle investment strategy only provide slightly higher probabilities to exceed a minimum RWR threshold, the growth TRF strategy provides significantly higher expected wealth outcomes and also higher probabilities with regards to exceeding most of the defined RWR thresholds. However, for extremely risk-averse superannuation members, the slightly lower tail risk exposure and, therefore, better outcomes for the portfolio in case of extreme events such as the recent global financial crisis (GFC) might still justify to invest in lifecycle or more conservative TRF strategies.

Our findings are relatively robust across different scenarios for market returns, contribution levels, sequencing risk and the initial balance of the superannuation portfolio 10 years before retirement. The effect of the initial accumulated wealth at the beginning of the last 10 years on the terminal wealth seems to be linear. We find that the effect of volatility and correlation regimes is very limited whereas the presence of one year of crisis (similar to the GFC) has a significant impact. We regards to sequencing risk, we find that the effect is higher if the crisis occurs at the beginning of the 10 year period as it prevents the wealth of the portfolio from taking off. After testing different contribution levels and salaries we find that increasing a mere 2.5% in the contribution level has significant impacts on the probability of reaching or exceeding the required amount for a comfortably lifestyle according to ASFA. Our findings suggest that low income investors should prefer the growth strategy to improve their chances of reaching a reasonably high balance at retirement.

In order to analyse different strategies and scenarios we consider both the individual assets as well as their dependence structure. The first models we consider are non-parametric and are based on historical simulation. For these models we examine different bootstrap and simulation methods. To analyse the individual assets we fit ARMA-GARCH models to account for the serial autocorrelation and the stochastic volatility. For the dependence structure amongst assets we consider a dynamic copula approach and fit several models that take into account the changing nature of the dependence structure between different asset classes.

Our parametric approach enables us to incorporate factors that affect the final wealth at retirement. With this approach we aim to provide an adequate analysis of their role. Dynamic copula models are far more flexible than other parametric models such as the Gaussian distribution. They are able to address the "heavy tailedness" of the data and the changing nature of the dependence structure of assets in times of economic stress. Examining different models for dependence, we find that the applied parametric models typically yield similar results. This also holds for the nonparametric models with the exception of the models that give higher weight to recent observations, which imply much more conservative expectations about the terminal wealth of the portfolio.

Overall, this paper helps to provide some guidelines for contributors on the role of key factors with regards to the value of superannuation portfolios at retirement age.

The remainder of the paper is divided into four sections. In Section 2 we discuss the applied methodology focusing on both parametric and non-parametric techniques. We review examples of copula families and describe the estimation of the applied copula. In Section 3 we describe the fit of the suggested models to the empirical data and provide results on a number of goodness of fit tests for the applied models. In Section 4 we analyse the performance of three different investment strategies as well as the impact of key factors on the distribution of terminal wealth outcomes for the considered portfolios. Finally, in Section 5 we conclude and suggest future directions for research.

2. Methodology

This section provides a brief review of approaches that will be used in the empirical analysis to model the dynamic behaviour of the returns of a superannuation portfolio. We distinguish between non-parametric and parametric approaches. For the former we apply a standard bootstrap technique, as well as the block bootstrap and the stationary bootstrap to simulate the joint returns of the asset classes. For the latter we suggest dynamic copulas in combination with ARMA-GARCH models for the marginal return series.

2.1. Nonparametric Approaches

The first modelling method selected in this study is a form of block bootstrap simulation, see Künsch (1989). To illustrate the rationale for this approach let us first consider a standard bootstrap resampling method. We consider monthly return data from January 1970 to December 2013. In a standard bootstrap the empirical monthly return vector of the asset classes in the sample are randomly resampled with replacement to generate asset class return vectors for the entire 10-year investment horizon. Since we randomly draw rows (representing months) from the matrix of asset class returns, we are able to retain cross-correlation between the asset class returns as given by the historical data series while assuming that returns for individual asset classes are independently distributed over time. Because the resampling is done with replacement, a particular data point from the original dataset can appear multiple times in a given bootstrap sample. This is particularly important in examining the probability distribution of future outcomes. For example, September, 1987 is the worst month for the stock market in our 44-year dataset. In this month the log-return from stocks was -54.7%, while bonds yielded a return of 1%. Although this is only one observation in 44 years worth of data, i.e. 528 monthly observations, a bootstrap sample of 10 years of monthly returns can include the return observation for September, 1987 several times in a sequence. Similarly, return observations for other months, good or bad, can also be repeated a number of times within a bootstrap sample. Because this method allows for inclusion of extreme return outcomes for a number of times in a particular simulated 10-year return path, a much wider range of outcomes is possible. We consider two approaches to this model. In the first approach we assume that future scenarios follow a uniform distribution from previous scenarios. In the second approach we assign different weights to the scenarios. Under this weighting scheme we let the probability of past returns decline exponentially as we go back in time. Under this model, given a sample of size n, sorted chronologically, the weight given to the i-th observation is

$$\frac{\lambda^{n-i}(1-\lambda)}{1-\lambda^n}, \tag{1}$$

where $0 < \lambda \leq 1$ is the weighting parameter. Note that the weight or probability assigned to the i-th return scenario is λ times the weight assigned to the $(i+1)$-th return. Also, when $\lambda \to 1$ the weights approach an equal value of $\frac{1}{n}$, for all scenarios, so this is equal to the first approach. Following similar studies we set $\lambda = 0.995$, see e.g. Hull (2012) and Shepperd (2013).

The asset class return vectors obtained by bootstrap resampling are combined with their respective weighs under each asset allocation strategy to generate portfolio returns for each month in the 44 year horizon. The simulation trial is iterated a large number of times for lifecycle strategies.

In spite of being able to capture cross-correlation, a major drawback of this method is its inability to capture autocorrelation. Because of this in a second model we consider a block bootstrap in which more than one element of the sample is taken to account for the serial autocorrelation. The block we consider is two years of data. This means that when a particular month is sampled a block of two years consisting of the returns for this month and the following 23 months is included, allowing autocorrelations for each asset class to be taken into account.

In these two historical simulation methods we assume that each of the elements of the sample has the same probability to be sampled. To complement this we also consider an approach with exponentially declining weights that assigns a higher probability to the more recent return observations. Models based on historical simulation have the advantage of letting the data speak for itself by avoiding a parametric assumption; however they do not allow for returns beyond a level that has been observed historically and, therefore, may underestimate future extreme events. To overcome this drawback, often a parametric model in combination with Monte Carlo simulation is used to generate simulation scenarios.

2.2. Parametric Approaches

Traditionally, parametric models for dependence have been based on the assumption of a multivariate distribution. This assumption restricts not only the dependence structure but also the marginal distributions. Further to this, these models generally rely on the correlation between the series to model dependence. The pitfalls of relying on the correlation coefficient only when measuring dependence have been extendedly reported in financial literature, see e.g. Embrechts et al. (1999).

As a response to this, copula functions have been used more recently to quantify dependence in risk related contexts. These models have the flexibility of modelling the dependence structure without restricting the marginals, allowing for a separation of the two, see e.g. Nelsen (2006). In more recent times dynamic copula models have emerged to also account for the changing nature of dependence structure through time, see e.g. Cherubini et al. (2012).

The use of copula and dynamic copula models have proven to be very effective in modelling dependence. However, they are based on the assumption of having an independent and identically distributed random sample, an assumption that is often violated. In order to overcome this limitation, the application of copulas in combination with models that also account for heteroscedasticity, such as ARCH and GARCH models, have been suggested, see e.g. Shams and Haghighi (2013). Our approach is based on a Dynamic Copula model with ARMA-GARCH innovations and will be described in more detail in the following section.

2.2.1. ARMA-GARCH- Models for Individual Assets

The first step in our model is to find an appropriate model for the marginals. Thus, we need to estimate the parameters for the conditional mean and conditional variance equations to account for their stochastic nature. We focus on different ARMA-GARCH specifications for each of the considered series and abstain from using additional exogenous variables. In order to avoid overfitting, the best model is chosen based on Akaike's Information Criterion (AIC) and the Ljung Box Test p-values. Our model provides more flexibility to modelling conditional correlations as it involves less complicated calculations in comparison to e.g. the GARCH-BEKK model by Engle and Kroner (1995). Considering their popularity in the economic and financial literature, we assume that the variance of the individual series of asset returns can be modelled by ARMA-GARCH models.

2.2.2. Dynamic Copula Models

A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959) but has only gained strong popularity for use in modelling financial or economic variables in the last two decades. For an introduction to copulas see e.g. Joe (1997) or Nelsen (2006), for applications to various issues in financial economics and econometrics, see, e.g. Cherubini et al. (2004), Frey and McNeil (2003), Patton (2006), just to name a few. As shown by Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al. (2006) and Luciano and Marena (2003), the use of correlation usually does not appropriately describe the dependence structure between financial assets and this could lead to inadequate risk measurement. Ang and Chen (2002) and Longin and Solnik (2001) empirically demonstrate that, in general, asset returns are more highly correlated during volatile markets and during market downturns. Dowd (2004) suggests that the strength of the copula framework is attributable to not requiring strong assumptions about the joint distributions of financial assets in a portfolio. Jondeau and Rockinger (2006) and Patton (2006) illustrate that copulas can be applied, not only directly to the observed return series but also, for example, to vectors of innovations after fitting univariate GARCH models to the individual return series. Overall, the use of copulas offers the advantage that the nature of dependence can be modelled in a more general setting than using linear dependence only. Copulas also provide a technique for decomposing a multivariate joint distribution into marginal distributions and an appropriate functional form for modelling the dependence between the asset returns.

In the following paragraphs we will briefly summarize the basic ideas and properties of copulas. For a definition of copulas we refer e.g. to Sklar (1959) or Joe (1997). Let X_1 and X_2 be continuous random variables with distribution functions F_1 and F_2 and a joint distribution function. Following Sklar (1959), there exists a function C such that

$$C(F_1(x_1), F_2(x_2)) = F(x_1, x_2). \tag{2}$$

This function C is called a copula and denotes a joint cumulative density function (CDF) of the independent, $U\tilde{}[0;1]$ distribution functions. Moreover, if the marginal distributions F_1 and F_2 are continuous, the copula function C is unique, see Sklar (1959), and the copula is an indicator of the dependence between the variables X_1 and X_2.

The literature suggests a wide range of different copulas, see, e.g., Joe (1997) or Nelsen (2006) for an overview of the most common parametric families of copulas. In the following we will limit ourselves to a description of a number of copula families that will be used further on in the empirical analysis. Among the most commonly used copulas in finance are the Gaussian, Student t, Clayton and Gumbel copula.

Probably the most intensively applied copulas in financial applications are the Gaussian and Student t copula. The Gaussian copula is constructed using a multivariate normal distribution and is defined as

$$C(u_1, u_2) = \int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{|R|}} \exp\left(-\frac{\mathbf{x}'R^{-1}\mathbf{x}}{2}\right) d\mathbf{x}, \tag{3}$$

where Φ denotes the standard univariate Gaussian distribution, $\mathbf{u} = (u_1, ..., u_2)$ and $\mathbf{x} = (x_1, ..., x_2)'$. The normal copula correlates the random variables rather near the mean and not in the tails. Therefore, it fails to incorporate tail dependence which can often be observed in financial data. In order to add more dependence in the tails, alternatively, the Student t-copula can be applied. The Student t copula is well known as accounting for stylised facts such as fat tails and the presence of tail dependence, see Joe (1997). The Student t copula with ν degrees of freedom and correlation matrix R is expressed in terms of integrals of its corresponding density $t_{\nu,R}$.

$$C(u_1, u_2) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \int_{-\infty}^{t_\nu^{-1}(u_2)} \frac{\Gamma\left(\frac{\nu+2}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\pi\nu\sqrt{|R|}} \left(1 + \frac{\mathbf{x}'R^{-1}\mathbf{x}}{\nu}\right)^{-\frac{\nu+2}{2}} d\mathbf{x}, \tag{4}$$

where t_ν denotes the Student t distribution with ν degrees of freedom.

Both the Gaussian and Student t copula are symmetric. However, often financial variables are observed to exhibit tail-dependence in only one of the tails, either the upper right or lower left edge of the data. For example, tail-dependence in the lower left tail indicates that the two variables have a tendency to simultaneously yield high negative returns. However, in situations where returns from one of the variables are highly positive the other financial variable may not be affected to the same extent. To model asymmetric tail-dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al. (2004). In this work we use two of the most prominent members of the family of Archimedean copulas, the Clayton and Gumbel copula,. The Clayton copula is defined as

$$C_\theta(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{\frac{-1}{\theta}},$$

$\theta > 0$.

The Gumbel copula is defined as

$$C_\theta(u_1, u_2) = \exp\left(-[(-\log u_1)^\theta + (-\log u_2)^\theta]^{\frac{1}{\theta}}\right), \tag{5}$$

$\theta > 1$. These copulas are both asymetric. Note that the higher the value of parameter , the greater is the degree of dependence between the considered variables. For further properties and examples of elliptical and Archimedean copulas and the construction of such copulas by using generator functions, we refer to Cherubini et al. (2004) or Nelsen (2006).

Note that due to the possible heteroskedastic behaviour of the return series, in the empirical analysis we will not apply copula models to the observed returns directly. Instead the copula functions will be estimated using the vectors of innovations after fitting univariate ARMA-GARCH models to the individual return series e.g. Jondeau and Rockinger (2006) and Patton (2006).

After fitting the stochastic process for the marginal return series, a dynamic conditional copula model can be estimated to specify the dynamics of the copula dependence parameter. Patton (2006) proposes observation-driven copula models where the time-varying dependence parameter is a parametric function of transformations of the lagged data and an autoregressive term. Then, using the marginal distribution of the standardized residuals, the dynamics of the parameters for the Gaussian, Student's t, Gumbel or Clayton copula can be specified. For the dynamics of the correlation for the Gaussian and Student t copula, following Patton (2006), we apply the following model:

$$\rho_t = \Lambda_1 \left\{ \omega + \beta\rho_{t-1} + \alpha \frac{1}{12} \sum_{j=1}^{12} F^{-1}(u_{t-j}) F^{-1}(v_{t-j}) \right\}, \tag{6}$$

with link function

$$\Lambda_1(x) = \frac{(1 - e^{-x})}{(1 + e^{-x})}$$

which ensures that the estimated correlation parameter ρ_t remains in its domain $(-1, 1)$. In a similar manner, the model for the two Archimedean copulas can be specified as:

$$\theta_t = \Lambda_2 \left\{ \omega + \beta\tau^U_{t-1} + \alpha \frac{1}{12} \sum_{j=1}^{12} |u_{t-j} - v_{t-j}| \right\}, \tag{7}$$

with link functions

$$\Lambda_2(x) = e^x$$

for the Clayton copula ($\theta > 0$) and

$$\Lambda_2(x) = e^x + 1$$

for the Gumbel copula ($\theta > 1$). The two transformation functions are also chosen to guarantee that the copula dependence parameter for the Clayton and Gumbel copula always remains in its domain.

Note that in this specification, the previous value of the parameter is used as a regressor to capture the persistence in the dependence parameter, while the mean of the last 12 observations of the transformed variables and , previous observations are used to capture any variation in dependence between the innovation series. The suggested dynamic copula models can then be estimated using maximum likelihood. In the following section we use the methodology described so far to model an empirical application.

3. Fitting the models to empirical data

In this section we fit the suggested parametric models to monthly log-returns from Australian equity and bond indices.

3.1. The Data

In order to set up a comprehensive analysis of different superannuation investment strategies, it is of great importance to identify the most important assets used to construct the different portfolios. By having a simple portfolios of representative assets it is possible to focus on the main factors that affect the final wealth at retirement. Because of this, for our empirical analysis we consider a portfolio consisting of Australian stocks and bonds. We employ Australian asset class monthly returns for the period January 1970 to December 2013. The Australian All Ordinaries Accumulation Index (AOI) is used as the proxy for broad equity returns. The proxy for Australian bonds is a spliced time series of three data sources, namely, the Andex Bond Accumulation Index from January 1970 to December 1976, the CBA Australian Bonds $>$ 10 Year Accumulation Index from January 1977 to September 1989, and the UBS Australia Composite Bond Index from October 1989 to December 2013. In our stylized analysis, these two assets classes are considered as the main constituents of MySuper investment strategies.

Series	Mean	Median	St. Dev.	Min.	Max.	Skew.	Kurtosis
Stocks	0.009	0.013	0.054	-0.547	0.173	-2.217	24.078
Bonds	0.007	0.007	0.019	-0.109	0.136	-0.171	12.585

Table 1: Descriptive Statistics for Logarithmic Returns of Australian Stocks and Bonds from January 1970 to December 2013

We are in particularly interested in appropriately modeling the dynamics of the individual return series, but also the dependence structure between returns from Australian stock and bond markets. For our analysis we consider logarithmic returns calculated from the original price series.[2] Table 1 provides descriptive statistics for the return series of the stocks and bonds. We consider the mean, median, standard deviation, minimum, maximum, skewness and kurtosis of the returns. The reported descriptive statistics suggest that mean returns, but also the standard deviation as well as skewness and kurtosis of the returns are significantly higher for the equity index in comparison to bonds. Figure 1 provides a plot of the cumulative performance of the stock index versus the considered bond index for the entire sample period. To make it easier to compare the performance of the two asset classes, each series is set equal to a base value of 100 at the start of the sample period in January 1970. The figure illustrates the significantly higher growth of the equity index in comparison to the bond index. However, it also indicates the significant drop of the AOI in 1987 and during the GFC from 2007-2009. We find that while the bond index yields a much lower performance in comparison to equity when considering the entire sample period. On the other hand it does not provide periods of extreme losses as the equity and the performance is clearly less volatile. However, the figure also illustrates that there were periods of relatively high volatility also in the Australian bond market and that investments into bonds cannot be considered as risk-free.

The correlation between the two assets for the entire sample period is approximately 0.25, suggesting at least a certain level of dependence between Australian equity and bond returns. We will investigate the dynamics of the dependence structure between the two assets classes in more detail later on.

3.2. Modelling the Marginals

To analyse the individual monthly return series we implement a two-stage procedure. In the first stage, we fit ARMA-GARCH models to each series and obtain the standardized residuals for each series. These residuals are then assumed to be independent and identically distributed

[2]Note that in the following we will refer to the calculated logarithmic returns simply as returns.

and the generalised inverse distribution is then used to generate a uniformly distributed sample, suitable for the copula analysis. Note that one could also model the dependence structure using the original return series. However, due to the heteroscedastic behaviour of financial returns, a conditional approach that models the dependence structure after filtering out autoregressive and heteroscedastic behaviour seems more appropriate as suggested by e.g. Grégoire et al. (2008), Jondeau and Rockinger (2006) and Patton (2006).

3.2.1. Estimation results for the ARMA-GARCH models

We focus on different ARMA-GARCH specifications for each of the considered series and abstain from using additional exogenous variables. In order to avoid overfitting, the best model is chosen based on the Akaike's Information Criterion (AIC) and the Ljung-Box Test (LBT) p-value. The LBT examines whether any of a group of autocorrelations of a time series are different from zero. It is a so-called portmanteau tes, i.e.instead of testing randomness at each distinct lag, it tests the 'overall randomness of the data based on a number of lags. The null hypothesis of the test is that data are independently distributed versus the alternative hypothesis that the data exhibit serial correlation. We decided to conduct LBTs for a maximum number of lags of $h = 5$, $h = 10$ and $h = 15$.

The optimal models suggested by AIC are an AR(1)-GARCH(2,1) model for the considered equity index and an ARMA(1,1)-GARCH(2,1) model for the bond index. As suggested by Table 2 conducted LBTs on the residuals after fitting the ARMA-GARCH models do not indicate significant levels of autocorrelation for the ARMA-GARCH filtered data. I a next step, the standardized residuals will then be used to model the dependence structure between the two asset classes by using copulas.

Time Series	**Chosen model**	**LBQ test p-value of residuals**		
		lags = 5	**lags = 10**	**lags = 15**
Stocks	AR(1)-GARCH(2, 1)	0.933	0.780	0.646
Bonds	ARMA(1, 1)-GARCH(2, 1)	0.741	0.834	0.586

Table 2: p-values of the LBQ test of residuals for different lags of the ARMA-GARCH models. The choice is based on LBQ and AIC tests

3.3. Estimation Results for the Copula Functions

In a next stage we investigate the dependence structure between the returns from the considered Australian equity and bond indices. We fit the Gaussian, Student t, Gumbel and

Clayton copula to the standardised residuals and estimate the dependence parameters for these copulas. Note that for the Student t copula also the degrees of freedom parameter needs to be estimated.

3.3.1. Results for Static Copula Models

One of the challenges is deciding on which copula provides the best fit to the actual dependence structure of the data. The literature suggests that information criteria such as e.g. AIC are generally not sufficient to provide enough understanding about the power of the decision rule employed, see e.g. Genest et al. (2006, 2009). Instead, goodness-of-fit (GOF) approaches are more powerful in deciding whether to reject or accept parametric copulas, making them the preferred choice in empirical applications. Therefore, in our empirical analysis, for selecting the most appropriate among a set of copulas, we decided to use goodness-of-fit tests that investigate the distance between the estimated and the so-called empirical copula (Genest et al., 2006, 2009). The empirical copula basically represents an observed frequency and is calculated from the empirical margins. It was originally introduced by Deheuvels (1979) under the name of empirical dependence function and is defined as

$$C_n^e(u_1, u_2) = F_n(F_{1,n}^{\leftarrow}(u_1), F_{2,n}^{\leftarrow}(u_2)),$$

whiere $\leftarrow$ denotes the generalised inverse function. Note that the empirical marginal distribution converges towards the actual distribution function for approaching infinity. The empirical copula is a consistent estimator of the true copula and, thus, is a well-accepted benchmark for copula goodness-of-fit tests.

The distance between the estimated and empirical copula is then evaluated using the Cramer-Von Mises (CVM) distance, and the parametric copula closest to the empirical copula is considered to provide the most appropriate fit to the dependence structure between the time series. Genest et al. (2009) provide various options for such tests by conducting a large Monte Carlo experiment and report particularly good results for the blanket tests using ranks or the Rosenblatt transform, recommending the CvM statistic as a distance measure.

For the bivariate case, the test procedure can be summarized as follows: consider a sample $(u_{1,i}, u_{2,i})$ for $i = 1...n$ and a parametric copula C_θ, the statistic is defined as:

$$\sum_{i=1}^{n} \left(C_n^e(u_{1,i}, u_{2,i}) - C_\theta(u_{1,i}, u_{2,i})\right)^2$$

1. Based on the empirical CDFs for the filtered marginal series, estimate the empirical copula and the parametric copula.

2. Using the Cramer-Von Mises statistic, calculate the distance between the empirical and the estimated copula.

In a first step we investigate the performance of static copula models, i.e. we assume that the copula dependence parameter is constant and fit the Gaussian, Student t, Gumbel and Clayton copula to the standardized residuals. According to Table 3, the CvM distance suggests the best fit to the data for the static Gaussian and Gumbel copula, while the Clayton copula with negative tail dependence provides clearly the worst fit. Surprisingly, these results suggest that monthly returns from the AOI and the considered bond index rather exhibit tail dependence in the upper right tail, i.e. when returns are highly positive for both asset classes. Typically the literature rather suggests tail dependence for financial assets in the lower left tail, i.e. during periods of market turmoils or financial crises. However, stocks and bonds are very different asset classes and one would not necessarily expect that returns from these assets co-move or show similar behavior under different market scenarios.

3.3.2. Results for Dynamic Copula Models

We also investigate the performance of dynamic copula models with a time-varying dependence parameter as suggested in the previous section. To investigate whether a static or dynamic copula model is more appropriate, we also compare the fit of these models. Given that we model the dependence structure between Australian equity and bond returns over a period of more than 40 years, one could expect to find various regimes of dependence between the two asset classes. Table 4 provides the results for both static and dynamic models for the applied Gaussian, Student t, Clayton and Gumbel copula ranked by their copula negative log-likelihood. We find that the models allowing for time-varying copula parameters outperform all static models. With respect to the best overall fit, the symmetric Gaussian and Student t copulas perform best, followed by the Gumbel copula. Note, however, that based on the log-likelihood measure the fit of the time-varying Gumbel copula is only insignificantly better than that of the Clayton copula.

To further illustrate the nature of dependence between the considered asset classes, in the following we examine the estimated time-varying parameters for the copula models. Let us first consider Figures 2 and 3, where a plot of the estimated static and time-varying parameters for the Gaussian and Gumbel copula is provided. The fixed parameter is estimated by simply

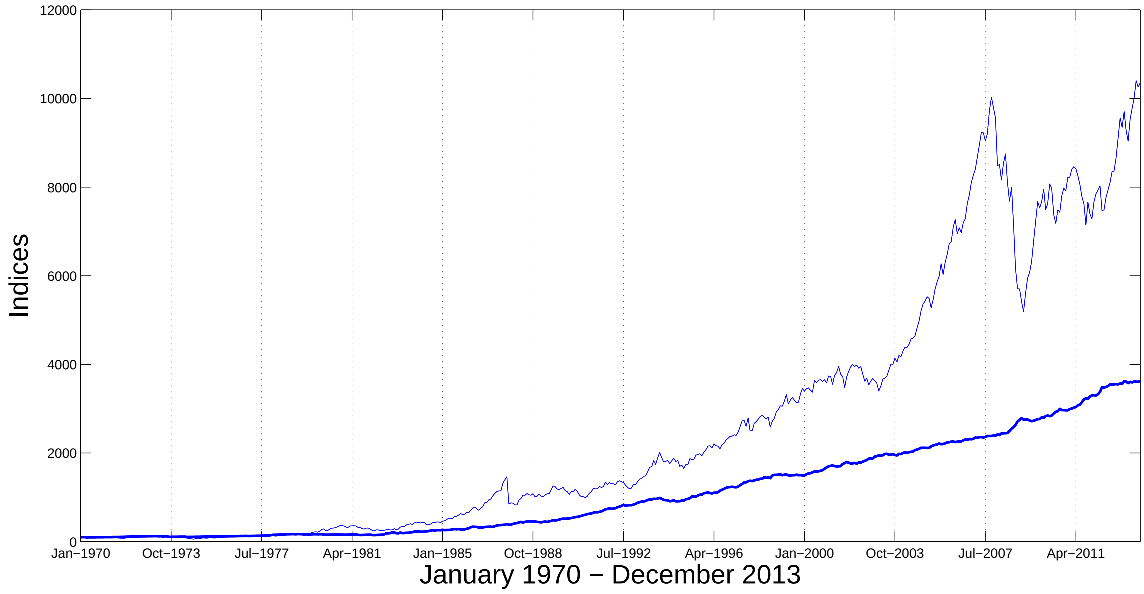

Figure 1: Cumulative performance of the Australian All Ordinaries Accumulation Index (AOI) and the proxy for the performance of Australian bonds for the sample period from January 1970 - December 2013. To make it easier to compare the performance of the two asset classes, each series is set equal to a base value of 100 at the start of the sample period in January 1970.

Copula Model	Cramer Von-Mises Statistic
Gaussian	0.0358
Student t	0.0404
Clayton	0.0476
Gumbel	0.0292

Table 3: Cramer-Von Mises statistic for different copula models

Copula Model	Copula Negative log-likelihood
Gaussian copula with time-varying parameter	-32.576
Student t copula with time-varying parameter	-29.045
Gumbel copula with time-varying parameter	-8.936
Clayton copula with time-varying parameter	-8.902
Student t copula with fixed parameter	-7.128
Gaussian copula with fixed parameter	-6.283
Gumbel copula with fixed parameter	-6.133
Clayton copula with fixed parameter	-4.993

Table 4: Negative log-likelihood for different copula models

fitting the copulas to the dependence structure between the standardized residuals of the applied ARMA-GARCH time series models. The time-varying dependence parameters are derived by fitting model (6) and (7) to the standardized residuals. We decided to choose a window length of 12 observations that corresponds to one year. Thus, the first twelve month period considers returns from February, 1970 to January, 1971, while the last window uses data from January to December, 2013.

The figures illustrate that there seems to be time-variation in the dependence structure between returns from Australian equity and fixed income markets. Figure 2 suggests that between 1970 and 2000 the correlation between monthly equity and bond returns was positive, ranging roughly from 0.2 up to 0.5. However, the figure illustrates a clear structural break in the early 2000s, and since then the dependence has decreased significantly and is typically negative. Overall, the estimated dynamic copula model yields a relatively smooth behaviour for the dependence parameter for the Gaussian copula. The estimated static copula correlation parameter is 0.17. Note however, that a static estimate would clearly underestimate the actual correlation between the returns for the period 1970-2000, while it would also not capture the recent negative correlation between equity an bond returns in Australian markets.

Considering Figure 3, we find a higher degree of time-variation for the parameter of the Gumbel copula. We observe several lapses of peaks and troughs around 1.15, while the behavior of the estimated parameter is not nearly as smooth as for the Gaussian copula. Our results indicate that joint upward movements of the two return series occur quite often during the considered sample period. Periods where the parameter of the Gumbel copula is approximately one indicate that the dependence is very weak. Similar to the behavior for the Gaussian copula, overall, the dependence in the upper tail also seems to be reduced from 2000 onwards, however, there is still a short period of an increase in upper tail dependence between the returns in the early 2000s. Note that conclusions as to whether there is a structural break or a significant change in the dependence structure during the considered period require further statistical tests as suggested by Patton (2006), that are beyond the objectives of this paper.

Figure 4 provides a plot of the stock index against the corresponding estimated volatility in Australian equity markets obtained by using the ARMA-GARCH models, and the correlation of the time-varying copula model for the considered sample period. The figure illustrates the significant changes in equity volatility throughout the sample period as well as the time-varying nature of dependence between the two asset classes. An appropriate parametric model needs

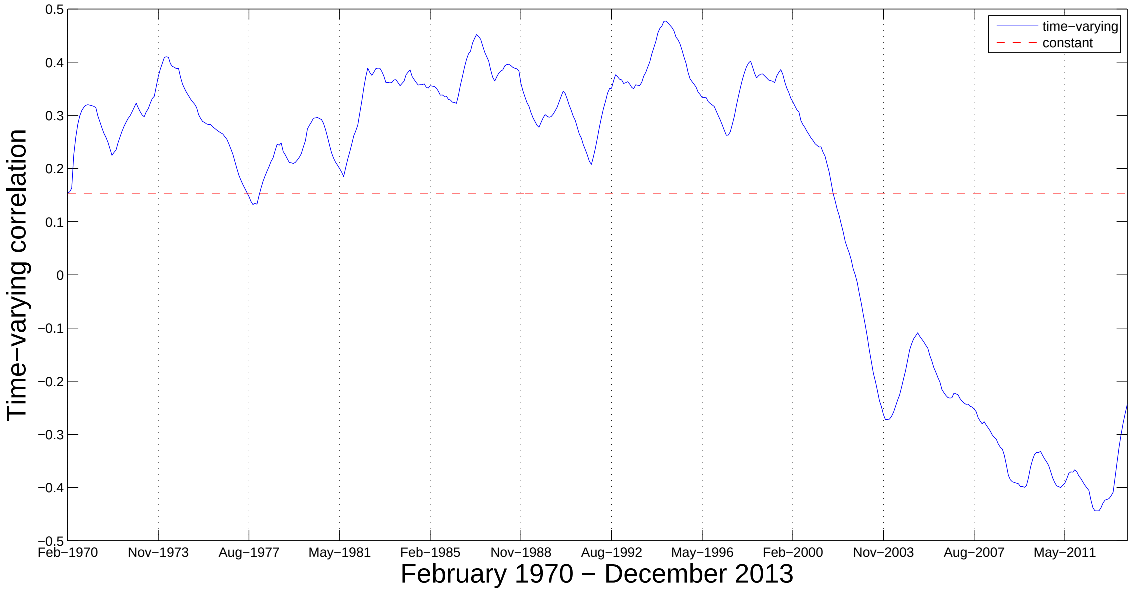

Figure 2: Moving parameter of Gaussian copula using Patton's model

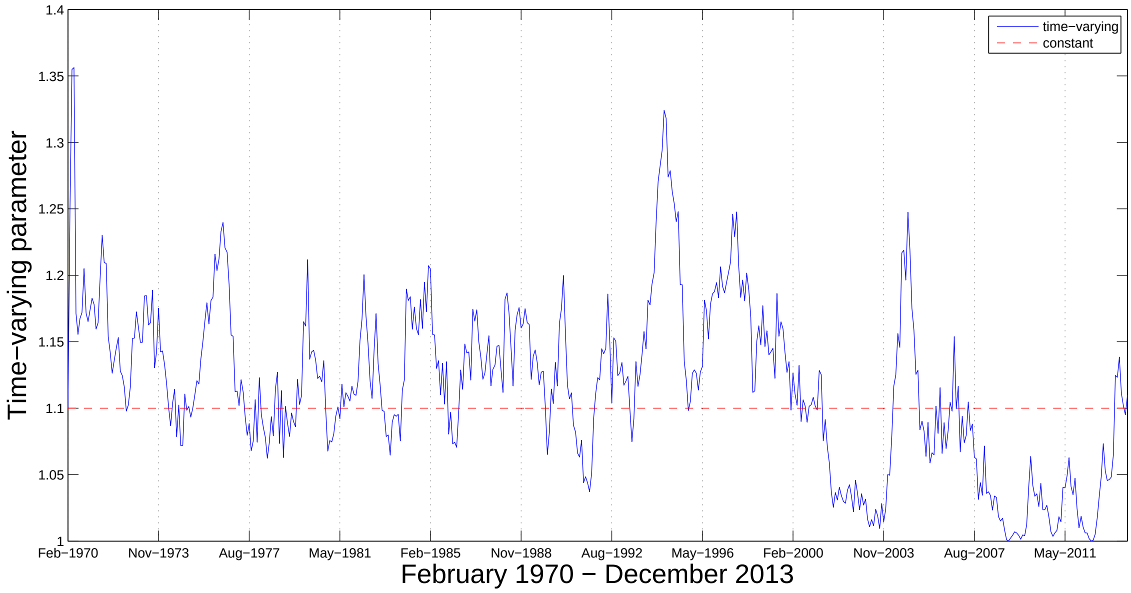

Figure 3: Moving parameter of Gumbel copula using Patton's model

to take these two features into account in order to realistically replicate the behavior of the individual asset classes as well as the co-movement of equity and bond returns. Therefore, the applied ARMA-GARCH models for the marginal return series in combination with a dynamic copula model for the dependence structure is clearly superior to a simple multivariate normal distribution. The latter would ignore both the heteroscedastic behaviour of the returns and the time-varying nature of the dependence.

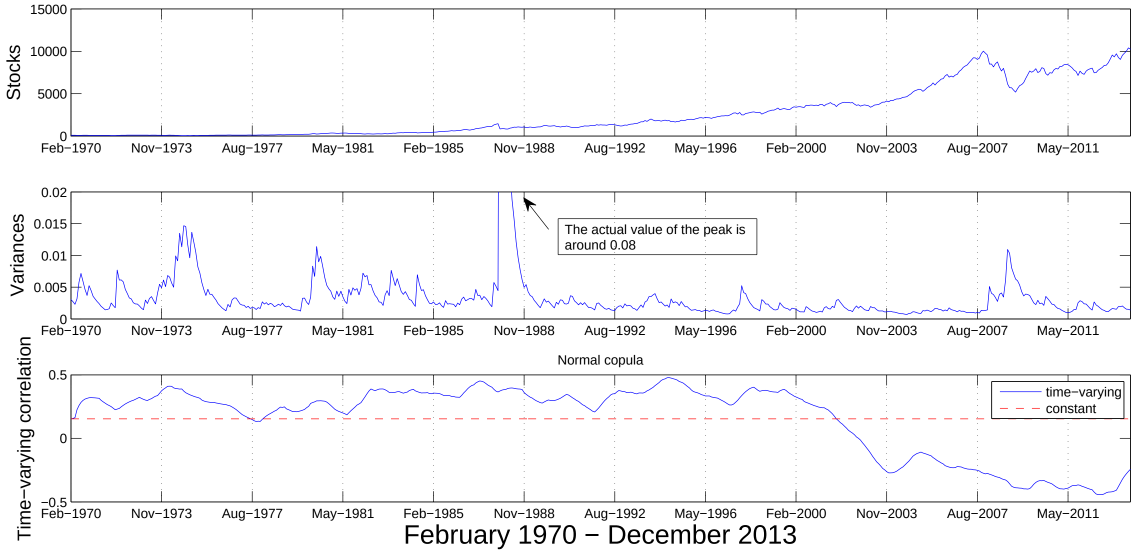

Figure 4: Performance of the considered Australian equity index (*upper panel*), estimated volatility in the Australian equity market (*middle panel*) and estimated time-varying copula correlation between equity and bond returns (*lower panel*).

4. MySuper Strategies and Performance Measures

In this section we review the MySuper investment strategies that will be examined in the empirical analysis. We also illustrate the applied performance evaluation measures, including retirement wealth ratios as well as risk-adjusted measures such as exceedance probabilities for wealth outcome thresholds. Finally, we illustrate the applied simulation procedure to generate distributions for the terminal values of superannuation portfolios.

4.1. Target Risk and Target Date Funds

There are a myriad of asset allocation approaches currently implemented in approved MySuper products. As discussed in previous sections, our analysis mainly focuses on the performance of superannuation portfolios during the last 10 years prior to retirement. Balanced or target risk funds (TRFs) maintain the same level of risk through time by holding a constant proportion of growth and defensive assets. TRFs are commonly employed in MySuper products at varying proportions of growth and defensive assets. TRFs strategies can range from 100% stocks to 100% bonds, while typically a *balanced* MySuper product has invested approximately 70% in growth and 30% in defensive assets, see, e.g., Chant et al (2014).

Further to TRFs we consider lifecycle or target date funds (TDFs), which recently have gained popularity in the MySuper universe. Typically, TDFs switch from growth to defensive assets according to a pre-determined glide-path as a worker approaches retirement. TDFs change the proportion of growth assets in the retirement portfolio as the worker approaches a retirement date using deterministic switching rules. TDFs have become a core product for investors saving for retirement, particularly in the U.S.

Overall, we analyse three examples of asset allocation for MySuper products. Two different TRFs and a TDF with a deterministic glide path from a balanced to a more conservative exposure.

1) A portfolio of 70% growth assets (equity) and 30% conservative assets (bonds) as an example for a typical Mysuper *balanced product*.

2) A portfolio of 30% growth assets (equity) and 70% conservative assets (bonds) as an example for a typical Mysuper *conservative investment product*.

3) A portfolio that linearly switches from 59% investment in equities to 37% investment in equities, while the remaining fraction is invested in bonds.

Note that for replicating a typical TDF strategy, we use data provided by Chant et al (2014) on glide paths of 23 MySuper lifecycle funds. Based on these results, a superannuation portfolio that linearly switches from 59% to 37% investment in equities over the last 10 years prior to retirement, provides a good approximation of the average glide path of Australian MySuper lifecycle products.

In the following we will analyze the performance of these three exemplary MySuper products across various performance criteria.

4.2. The Retirement Wealth Ratio for MySuper Portfolios

To evaluate asset allocation strategies and assess their appropriateness as default investment options in MySuper strategies, we need to make plausible assumptions about the rationale that may guide the selection of a specific asset allocation strategy as a default option from many competing candidates. The basic motivation behind instituting retirement savings plans is to generate adequate income for the participating employees after retirement. In that case, the performance of MySuper strategies should be measured in terms of their ability to generate sufficient retirement income (Baker et al., 2005). Therefore, the principal investment objective of such plans will be to maximize the terminal value of the portfolio at the point of retirement. The terminal value will directly determine the amount of annuity the retiring employees are able to purchase for sustenance during post-retirement life. Past studies have often focused on the absolute value of the participant's accumulated assets at retirement. Instead we employ a ratio which compares the terminal wealth of the participant's retirement account to their terminal income. The rationale for such an analysis is that a participant's post-retirement income expectations are closely linked to their immediate income before retirement, see, e.g Basu and Drew (2009). Therefore, the so-called retirement wealth ratio (RWR) that is defined as the wealth at retirement divided by the final yearly income might provide a more reasonable measure to examine the performance instead of considering the absolute terminal value of the portfolio only.

Note, however, that higher mean or median values of RWR outcomes for an investment strategy do not necessarily imply that the strategy is superior. The trustees also need to consider the risk associated with an investment strategy, since participants would want a better exploitation of trade-off between risk and reward. Therefore, in our analysis we examine the entire distribution of RWR outcomes and also focus on lower or higher percentiles of the RWR distribution as well as on the probability of exceeding thresholds of RWR outcomes. Hereby, we assume that the ultimate goal of a MySuper strategy is to attain a specific amount of wealth relative to their terminal income. The investment risk most relevant to participants might be the failure to generate a minimum terminal retirement wealth ratio (TRWR). The literature does not provide clear guidance on an adequate TRWR, such that we decide to examine the probabilities of exceeding TRWRs of 5, 8 and 10. These thresholds correspond to a terminal wealth in a superannuation account that is equal to five times, respectively eight or ten times, the final annual salary of an investor.

Finally, to complement this analysis we also consider a fixed amount for the wealth at retirement which is not dependent on the person's income. In order to do this, we use the amount that is required for a *comfortable standard of living* proposed by the Association of Superannuation Funds of Australia (ASFA, 2014). This amount is $430,000 and we use an annual average weekly earnings (AWE) growth rate of 3.75% as a deflator. [3]

[3]Note that ASFA also considers an amount that is sufficient for a *modest standard of living*, given the pension supplements. However, we decided to focus on the *comfortable standard of living* amount only in our analysis.

4.3. Simulation procedure

In order to assess the effectiveness of the examined MySuper strategies in achieving TRWRs, we follow a simulation procedure considering four parametric models as well as four bootstrap approaches based on historical simulations that have been described in Section 2. For the parametric techniques, we decided to apply the estimated ARMA-GARCH models in combination with either the Gumbel or the Gaussian copula for the dependence structure between equity and bond returns. Recall that the Gumbel copula yielded the best results for the static case, assuming a constant parameter for the dependence structure over the sample period, while the Gaussian copula model with a time-varying dependence parameter yielded the overall best fit to the data. While the application of nonparametric approaches for the simulation of equity and bond returns is relatively straightforward and has been described in Section 2, for the parametric models the following algorithm is applied to generate time series of returns for the considered equity and bond index:

1) *Model Estimation*

   Using monthly logarithmic returns of the considered equity and bond index for the sample period from January 1970-December 2013, we fit ARMA-GARCH models (Table 2) to the marginal return series. We then fit the static and dynamic Gaussian and Gumbel copula models to the standardised residuals.

2) *Simulation of Standardized Residuals*

   2a) Static Copula Models: Assuming a fixed copula dependence parameter θ, we generate 10,000 bivariate samples of size 120 for the standardized residuals, corresponding to 10 years of monthly observations.

   2b) Dynamic Copula Models: In the case of dynamic copula models, we initialize the simulation process by generating the first element of the bivariate standardized residuals for time $t = 1$, using the most recent estimate of the time-varying copula dependence parameter θ_0. Then we use equation (6), (7) respectively, to update the copula dependence parameter estimate θ_1. Based on the updated copula dependence parameter the next pair of standardized residuals is created, and so on. Overall, we simulate 10,000 bivariate series of size 120.[4]

[4] Note that the simulation from the dynamic copula model with time-varying copula parameters is far more computationally intensive than for the static version of the model.

3) We filter the corresponding samples through the ARMA-GARCH models to generate 10,000 random samples for the time-series of 120 logarithmic returns which we then convert to discrete returns to use them for the conducted analysis.

5. Empirical Results

As mentioned earlier in this section the main interest in our analysis is to determine what factors affect whether or not the retiree has an adequate wealth at retirement. For this purpose we mainly focus on the suggested RWR, however, we will also include results for the analysis on exceeding the *comfortable standard of living* proposed by ASFA (2014). Using samples of 10,000 different wealth outcomes for each simulation method, we are able to obtain several results involving the RWR for different models and factors. In particular, we estimate the probability of achieving or exceeding different TRWRs for the MySuper strategies as well as their different quantiles. We first consider a base scenario that is meant to represent the average income and contribution of a MySuper investor. After this we consider different models and the impact of a range of factors that affect terminal wealth outcomes for an investor. Among other factors, we consider different market regimes of volatility and return for the asset classes, sequencing risk, different types of the investor's contributions as well as the weight that is given to more recent observations in the simulation procedure, .

5.1. Base scenario

In order to conduct the empirical analysis, in a first step we consider a representative investor 10 years before retirement. We assume that the investor has an annual income of approximately \$63,150, a superannuation contribution of 9.5% what corresponds to a monthly contribution of \$500. We further assume that the income increases by 4% annually and that the current balance of the superannuation investor is \$250,000. Considering these values, the terminal yearly income is \$93,489. This means that, for example, to exceed a TRWR of 5, the terminal wealth of the portfolio at retirement must be at least \$467,445, while to exceed a TRWR of 8 the value of the portfolio is required to be greater than \$747,912. Note that the ASFA comfortable standard of living corresponds to a balance of at least \$621,369 in 10 years time, corresponding approximately to a RWR of 6.65 for the considered investor.

In Table 5 we present the mean as well as different quantiles of the RWR for the different strategies considering three representative models that include one parametric and two non-parametric approaches. In particular we present results for the Gaussian copula model with

MODEL	Strategy	Mean RWR	Quantile for RWR						
			1%	**5%**	**10%**	**50%**	**90%**	**95%**	**99%**
Gaussian Copula with time-varying parameter	Balanced	**9.8401**	1.9979	4.1824	5.2853	9.1022	14.6848	17.5797	26.7048
	Conservative	**8.2407**	2.9342	4.7010	5.5175	7.9255	11.0678	12.5587	17.3262
	Lifecycle (TDF)	**8.9770**	2.7758	4.5925	5.4962	8.4878	12.6497	14.3637	21.8208
Block bootstrap with equal weights (block size n = 6)	Balanced	**9.6308**	3.3949	4.4004	5.3297	8.9881	14.9016	17.0258	22.6976
	Conservative	**8.3041**	4.7262	5.4712	5.9489	8.0821	10.9521	11.8340	13.9327
	Lifecycle (TDF)	**8.8513**	4.1927	5.1049	5.7699	8.5105	12.4031	13.8196	17.5419
Block bootstrap with declining weights (block size n = 6)	Balanced	**8.7346**	3.1767	4.3386	5.0390	8.1825	12.9661	15.1797	19.0349
	Conservative	**7.6719**	4.8623	5.4882	5.9126	7.5253	9.5770	10.3053	11.9993
	Lifecycle (TDF)	**8.1124**	3.9673	5.0773	5.5869	7.8803	10.8954	11.9847	14.8022

Table 5: Mean and quantiles of RWRs for the three strategies according to different models

RWR	5			6.65			8			10		
Strategy	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**
Gaussian Copula with time-varying parameter	0.912	0.933	0.934	0.789	0.758	0.798	0.641	0.480	0.582	0.400	0.179	0.288
Block bootstrap with equal weights (block size n = 6)	0.919	0.979	0.956	0.767	0.792	0.791	0.605	0.519	0.575	0.398	0.196	0.284
Block bootstrap with declining weights (block size n = 6)	0.907	0.984	0.955	0.723	0.740	0.731	0.524	0.379	0.477	0.295	0.069	0.169

Table 6: Probability of exceedance of different TRWRs for the three strategies according to different models

a time-varying copula dependence parameter, historical simulation using a block bootstrap approach with block size $n = 6$, and, the same block bootstrap approach with assigned weights according to formula 2.1 such that more recent return observations get a higher weight in the simulation procedure. We find that for all three simulation methods, the highest average RWR is obtained for the *balanced* strategy with the highest weight in equity in comparison to the two other strategies. The second highest average RWR is obtained for the lifecycle (TDF) strategy, while the conservative investment strategy with 70% in defensive assets clearly yields the lowest expected outcome for the RWR in all three cases. We also find that in terms of the mean outcomes for the different strategies, there is only a minor difference between the parametric Gaussian copula model and the standard block bootstrap with $n = 6$. The parametric approach and block bootstrap method suggest a mean RWR of approximately 9.84, respectively 9.63 for the balanced MySuper strategy, while they also yield very similar results for the conservative strategy ($RWR = 8.24$ for the parametric and $RWR = 8.30$ for the block bootstrap) and the lifecycle strategy ($RWR = 8.98$, respectively, $RWR = 8.85$). Interestingly, the expected outcomes for the RWRs are significantly lower when a block bootstrap with declining weights for earlier observations is applied. Conducting the simulation procedure with assigned higher weights to more recent observations yields $RWR = 8.73$ for the balanced MySuper strategy, $RWR = 7.67$ for the conservative, and $RWR = 8.11$ for the lifecycle strategy. This makes sense given that in recent times returns have shown lower values with respect to previous times. This is an important result, as it indicates that if stock and bond returns behave more like they have done in recent times, lower RWR outcomes will be achieved. It implies that investors need to adjust their expectations, as this is fundamental for the planning of retirement outcomes.

Table 5 also provides information on higher and lower quantiles of the simulated RWR distribution. Such an analysis is also in particular useful for examining possible worst case scenarios for investors, which correspond to the 1%, 5% and 10% quantiles of the distribution. We find that for the lower tail of the distribution, the conservative strategy with a higher share in defensive assets provides the best results: for example, based on a block bootstrap with equal weights, the 1% quantile for the conservative strategy yields $RWR = 4.72$, while comparable values for the lifecycle and balanced strategy are $RWR = 4.19$, respectively $RWR = 3.39$. Independent of the chosen simulation technique, the conservative strategy always yields the best outcome for the 1%, 5% and 10% quantile of the distribution. However, the differences between the suggested strategies in the lower tail of the distribution are not that substantial. Even for the

10% quantile we see that the three strategies typically provide very similar results, while for the median it is always the balanced MySuper portfolio with the highest share in equity that yields the best results. With respect to higher quantiles of the RWR distribution, we find a clearly superior performance of the balanced portfolio over the two other strategies. For example, when considering the 95% quantile of the simulated distribution (parametric approach), we find that the balanced strategy offers a 5% chance of exceeding $RWR = 17.58$, while the corresponding figures are significantly lower, with $RWR = 12.55$, respectively $RWR = 14.36$ for the conservative and lifecycle strategy. This confirms that strategies with a higher share in growth assets can provide much higher upside potential in comparison to more conservative strategies also over the last 10 years. It also points towards the fact that, depending on how risk averse an investor is, she may or may not want to switch to more conservative strategies with limited upside potential and only slightly reduced tail risks.

Finally, we make an observation regarding the simulated distributions: it seems like the parametric approach yields a distribution with a significantly wider range in comparison to the non-parametric techniques. For example, for the balanced MySuper strategy, the spread between the 1% and 99% quantile of the distribution is approximately 24.70 for the parametric approach, while comparable figures are 19.30 for the block bootstrap with equal weights and 15.86 for the block bootstrap with declining weights. This emphasizes the difference between parametric and non-parametric simulation approaches: the former also allow for even more extreme outcomes that may have not been observed historically, while the latter is entirely based on historical return observations for the asset classes.

Table 6 presents the probabilities of exceeding the specific TRWR of 5, 6.65, 8 and 10 for the three simulation techniques. Clearly, the results are in line with our previous findings for the simulated distributions of RWRs. We find that for the lowest threshold $RWR = 5$, it is typically the conservative strategy that provides the highest exceedance probability. For the block bootstrap technique we obtain a probability of approximately 97.9% to exceed this threshold for the conservative strategy, while lower probabilities of 91.9% and 95.6% are obtained for the balanced and lifecyle (TDF) strategy. For $RWR = 6.65$, corresponding to the suggested ASFA comfortable standard of living, our results depend on the chosen simulation method: while for a parametric approach both the balanced and the lifecycle strategy yield higher probabilities for exceeding this threshold, for the non-parametric approach it is still the conservative strategy that yields the highest probability. As expected, for $RWR = 8$ and

$RWR = 10$, it is the balanced MySuper portfolio with the highest share in growth assets that yields the highest probability. For example, usig a parametric approach, the probability for a portfolio with RWR of 10 or higher is still approximately 40% for the balanced strategy, while it is 28.8% for the lifecycle and only 17.9% for the conservative strategy.

Similar to the results in Table 5, we find that the probabilities to achieve the specified TRWRs are generally lower, when the block bootstrap with declining weights is applied. The differences become more pronounced for higher thresholds such as e.g. $RWR = 8$ or $RWR = 10$. This emphaseizes the weaker performance of financial markets with lower upside potential in more recent years, in comparison to the entire 40 year period. These results are valid for across all implemented strategies and reinforces the fact that superannuation investors might have to expect lower outcomes for their portfolio if financial markets continue to provide returns that are similar towhat could be observed over the last decade.

Overall, as expected the probabilities of achieving or exceeding TRWRs are decreasing as the threshold gets higher. A fact that stands out from our analysis is also that the chosen investment strategy becomes increasingly important for high values of the TRWR. We find that investing in growth assets as we approach retirement highly increases the probabilitly of achieving high RWR. These results may not entirely be in line with general recommendations for lifecycle investment where switching to defensive assets is recommended for members that approach the age of retirement. Our results also indicate that for the chosen benchmark investor, it is relatively easy to reach the ASFA comfortable lifestyle.

5.2. Robustness check

We now focus on considering the impact of other factors in our analysis. We first consider the impact of the chosen simulation technique: for the parametric models we investigate impacts on the RWR with regards to the choice of the copula and whether a static or dynamic copula model is applied in the simulation procedure. For the non-parametric approaches, we examine the impact of the chosen block size as well as the weights that are given to more recent return observations in comparison to observations from earlier periods.

5.2.1. Parametric models

We first test the impact of the chosen copula model on the results by considering various parametric models. The additional models include the Gumbel copula with a fixed dependence parameter as well as the Student t and the Clayton copula with time-varying parameters. In

RWR	**5**			**6.65**			**8**			**10**		
Strategy	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**
Gaussian copula with fixed parameter	0.915	0.935	0.931	0.778	0.736	0.774	0.632	0.492	0.580	0.401	0.203	0.308
Gumbel copula with fixed parameter	0.916	0.940	0.934	0.790	0.745	0.783	0.630	0.491	0.573	0.380	0.186	0.278
Gaussian copula with time-varying parameter	0.912	0.933	0.934	0.789	0.758	0.798	0.641	0.480	0.582	0.400	0.179	0.288
Student t copula with time-varying parameter	0.912	0.926	0.926	0.776	0.751	0.777	0.630	0.468	0.560	0.391	0.167	0.275
Gumbel copula with time-varying parameter	0.925	0.944	0.942	0.797	0.760	0.798	0.634	0.487	0.577	0.396	0.186	0.283
Clayton copula with time-varying parameter	0.918	0.943	0.937	0.803	0.754	0.795	0.653	0.495	0.593	0.402	0.169	0.285

Table 7: Probability of exceedance of different TRWRs for the three strategies according to different parametric models

RWR	5			6.65			8			10		
Strategy	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**	**Bal**	**Cons**	**TDF**
Block bootstrap with block size n = 6	0.919	0.979	0.956	0.767	0.792	0.791	0.605	0.519	0.575	0.398	0.196	0.284
Stationary bootstrap average block size n = 6	0.905	0.961	0.942	0.732	0.753	0.749	0.580	0.487	0.559	0.379	0.173	0.284
Block bootstrap with block size n = 3	0.915	0.973	0.952	0.748	0.770	0.767	0.592	0.492	0.567	0.380	0.173	0.275
Block bootstrap with block size n = 12	0.910	0.966	0.947	0.758	0.787	0.783	0.602	0.521	0.586	0.406	0.202	0.321
Block bootstrap with block size n = 24	0.936	0.970	0.960	0.793	0.819	0.815	0.643	0.565	0.631	0.431	0.215	0.325
Block bootstrap (declining weights) with block size n = 6	0.907	0.984	0.955	0.723	0.740	0.731	0.524	0.379	0.477	0.295	0.069	0.169
Stationary bootstrap (declining weights) average block size n = 6	0.878	0.968	0.934	0.651	0.661	0.668	0.447	0.292	0.391	0.228	0.053	0.124
Block bootstrap (declining weights) with block size n = 3	0.917	0.986	0.962	0.740	0.760	0.756	0.552	0.378	0.486	0.300	0.054	0.158
Block bootstrap (declining weights) with block size n = 12	0.902	0.982	0.946	0.715	0.758	0.745	0.524	0.381	0.480	0.285	0.065	0.164
Block bootstrap (declining weights) with block size n = 24	0.918	0.982	0.958	0.724	0.740	0.743	0.531	0.384	0.479	0.279	0.085	0.164

Table 8: Probability of exceedance of different TRWRs for the three strategies according to different nonparametric models.

Table 7 we compare results for exceeding the specified $TRWR = 5, 6.65, 8, 10$ for the additional models to the parametric benchmark model reported earlier, i.e., the Gaussian copula with a time-varying dependence parameter. Interestingly, we find that all parametric models provide rather similar results with respect to the simulated exceedance probabilities for the defined thresholds. These findings also suggest that the choice of the parametric copula does not have a significant impact on the RWR outcomes for a superannuation investor.

5.2.2. Nonparametric models

For the nonparametric approaches, we compare the benchmark block bootstrap methods with block size $n = 6$ to an alternative stationary bootstrap method method with average block size of $n = 6$ and methods with alternative values for the blocksize of $n = 3$, $n = 12$ and $n = 24$. For each of these methods we also apply the scheme allowing for declining weights as outlined in equation (2.1). Table 8 presents the results for the different nonparametric models. We find that neither the applied block size nor whether a standard or a stationary bootstrap is applied seems to have very significant impacts on the results. Among the methods with equal weights for each observation, the stationary bootstrap seems to yield the lowest probabilites for exceeding the defined TRWRs, while the probabilities are the highest for a block bootstrap with $n = 24$. However, all five methods yield rather similar results.

On the other hand, all bootstrap methods with declining weights for more distant observations yield lower exceedance probabilities. While the effect is less pronounced for lower thresholds such as $RWR = 5$ and $RWR = 6.65$, it becomes more and more significant for higher thresholds. For example, a block bootstrap with block size $n = 24$ yields a probability of 32.5% to achieve or exceed an outcome of $RWR = 10$ when all observations are equally likely to enter the simulation, while the corresponding probability is only 16.4% for exponentially declining weights. This provides further evidence for our previous results pointing towards lower expectations of the terminal wealth outcomes, if higher weights are given to more recent observations. We will further investigate this in the following section.

5.3. Impacts of the considered historical time period

In the following we examine the impact of various factors on the terminal wealth outcomes for superannuation investors. The first part of our analysis aims to determine the impact of the considered historical time period. Given that the original period included 43 years of monthly observations for equity and bond returns, in the following we examine the results for shorter

Period	Series	Mean	Median	St. Dev.	Min.	Max.	Skew.	Kurtosis
Complete data	**Stocks**	0.009	0.013	0.054	-0.547	0.173	-2.217	24.078
	Bonds	0.007	0.007	0.019	-0.109	0.136	-0.171	12.585
Last 20 years	**Stocks**	0.007	0.014	0.038	-0.150	0.077	-0.947	4.287
	Bonds	0.005	0.005	0.010	-0.026	0.041	0.223	3.998
Last 10 years	**Stocks**	0.008	0.019	0.041	-0.150	0.077	-1.178	4.727
	Bonds	0.005	0.005	0.008	-0.012	0.030	0.506	3.430

Table 9: Descriptive statistics for logarithmic returns of the considered Australian stock and bond indices for different time periods of January 1970 - December 2013 (complete data), January 1994 - December 2013 (last 20 years) and January 2004 - December 2013 (last 10 years).

historical time periods, i.e. the last 10 and 20 years, respectively. Table 9 provides descriptive statistics for the considered monthly equity and bond returns for the entire sample period from January 1970 - December 2013 as well as sample periods covering the last 20 and the last 10 years. We find that average monthly equity and bond returns are higher for the entire 43 year sample in comparison to sample periods that cover the last 10 or 20 years only. However, we also observe a higher standard deviation of equity and bond returns for the entire sample period.

In Tables 10 and 11 we present the mean RWR outcome as well quantiles for the simulated RWR distribution based on using the last 20 years, respectively, 10 years of historical returns observations. The results confirm earlier findings on the impact of how recent and more distant returns are treated in the simualtion period. In comparison to simulation results that were based on the entire sample period, see Table 5, we obtain significantly lower outcomes for the RWR. For example, the parametric approach yields an average $RWR = 9.84$ for the balanced MySuper strategy, while we obtain $RWR = 8.90$ and $RWR = 9.47$ when the simulation is based on historical return observation spanning the last 20 years, respectively, the last 10 years. For the conservative MySuper strategy we obtain $RWR = 8.07$ (for 20 years of historical data) and $RWR = 7.89$ (for 10 years of historical data) in comparison to $RWR = 8.24$ for the entire sample period. Finally, the lifecycle TDF strategy yields $RWR = 8.42$ (20 years) and $RWR = 8.53$ (10 years), what i also significantly lower than the average $RWR = 8.98$ when the simulation is based on the last 43 years of data.

Overall, we find that the chosen historical sample period has significant impacts on the simulation results. This emphasizes that return expectations for superannuation investors will highly depend on the assumptions being made about the behaviour of future equity and bond returns. If returns are more likely to behave like in more recent times, expectations about the terminal wealth of superannuation investors need to be reduced.

MODEL	Strategy	MEAN RWR	Quantile for RWR						
			1%	**5%**	**10%**	**50%**	**90%**	**95%**	**99%**
Gaussian with time-varying parameter	Balanced	**8.9009**	4.3019	5.4070	5.9920	8.6851	12.1702	13.2606	15.6411
	Conservative	**8.0710**	5.5883	6.2174	6.5414	7.9769	9.6991	10.2042	11.4161
	Lifecycle (TDF)	**8.4284**	5.0109	5.8576	6.3600	8.3142	10.6725	11.3933	12.6834
Historical simulation with block bootstrap equal weights (n = 6)	Balanced	**8.8218**	4.0733	5.1148	5.7531	8.5293	12.2254	13.4946	16.4794
	Conservative	**7.9909**	5.8251	6.3433	6.7069	7.9500	9.3542	9.8023	10.6837
	Lifecycle (TDF)	**8.3355**	4.9273	5.8802	6.3269	8.2342	10.4885	11.2144	12.6670
Historical simulation with block bootstrap declining weights (n = 6)	Balanced	**8.7124**	3.6251	4.8683	5.5717	8.3507	12.2909	13.5435	15.9461
	Conservative	**7.8299**	5.6909	6.2750	6.6228	7.7883	9.0879	9.4630	10.2486
	Lifecycle (TDF)	**8.1934**	4.7399	5.7375	6.1804	8.0803	10.3720	10.9796	12.5394

Table 10: Mean and quantiles of the simulated RWR distribution for the three MySuper strategies based on using the last 20 years of historical returns observations for the parametric and non-parametric models.

MODEL	Strategy	MEAN RWR	Quantile for RWR						
			1%	**5%**	**10%**	**50%**	**90%**	**95%**	**99%**
Gaussian with time-varying parameter	Balanced	**9.4713**	3.6155	4.7560	5.5194	8.9470	14.1394	16.0798	19.4447
	Conservative	**7.8936**	5.0733	5.7978	6.2223	7.8259	9.6783	10.3501	11.6835
	Lifecycle (TDF)	**8.5389**	4.4550	5.3321	5.9602	8.2956	11.4752	12.5729	14.4426
Historical simulation with block bootstrap equal weights (n = 6)	Balance	**9.1802**	3.3787	4.4637	5.1934	8.6986	13.6542	15.2958	18.4410
	Conservative	**7.7905**	5.6372	6.2070	6.5224	7.7679	9.0968	9.4244	10.1604
	Lifecycle (TDF)	**8.3405**	4.4960	5.3383	5.9603	8.1858	10.9371	11.6182	13.4308
Historical simulation with block bootstrap declining weights (n = 6)	Balance	**8.8626**	3.4111	4.4389	5.0688	8.3525	13.4087	14.8579	18.1397
	Conservative	**7.6713**	5.6346	6.1485	6.4486	7.6356	8.9455	9.3309	10.0200
	Lifecycle (TDF)	**8.1558**	4.5739	5.3338	5.8239	7.9843	10.7288	11.5208	13.0482

Table 11: Mean and quantiles of the simulated RWR distribution for the three MySuper strategies based on using the last 10 years of historical returns observations for the parametric and non-parametric models.

5.4. Impact of different market regimes

We now use the estimated parametric Gaussian copula model to analyse different market regimes of volatility and correlation. We study four different regimes at the beginning of the 10 year simulation period: (i) a low volatility regime for the equity market, (ii) a high volatility regime for equity markets, (iii) a regime with low correlation between equity and bond returns, (iv) a high correlation regime.

The procedure for creating starting values for the low and high volatility regimes is the following: we sort the monthly observations based on estimated volatility according to the parametric model. Then we determine the 0.01 quantile (for the low volatility regime) and 0.99 quantile (for the high volatility regime) of the volatility distribution as starting values. To find the corresponding value of correlation between equity and bond returns for these regimes, we take the median correlation of the 10 months with volatility closest to the 0.01 quantile, respectively 0.99 quantile. These volatility-correlation combinations are then the starting values for the simulation from the parametric model

A similar procedure is conducted to determine the low and high correlation regimes: we sort the monthly observations based on the estimated time-varying copula correlation parameter. Then we determine the 0.01 quantile (for the low correlation regime) and 0.99 quantile (for the high correlation regime) of the correlation distribution as starting values. To create the corresponding value for volatility in these regimes, we take the median volatility of the 10 months with correlation closest to the 0.01 quantile or 0.99 quantile, respectively,.

Interestingly, as indicated by Table 12, the impact of the different regime during the first month of the simulated 10 year period is rather limited. The highest RWR outcomes are obtained in the case of a high volatility regime followed by a high correlation regime. The lowest RWR outcomes are achieved when an initially low volatility regime is assumed. As mentioned earlier the differences between the outcomes are not really significant. The lack of difference between them might be due to a typically rather quick adjustment of the market after a specific regime. Although autocorrelation between the returns is assumed, a specific volatility or correlation regime at the beginning of the simulation period does not seem to last for a very long time and has no significant impact over a 10 year simulation period.

5.5. The Impact of a market crash and sequencing risk

As we have seen in the previous subsection, the effect of regimes of volatilty and correlation on wealth outcomes at retirement age is rather limited. We now would like to examine the

Regime	MODEL	Strategy	MEAN RWR	Quantile for RWR						
				1%	5%	10%	50%	90%	95%	99%
Low volatility	**Gaussian with time-varying parameter**	Balanced	**9.962**	2.507	4.192	5.369	9.174	15.192	18.166	28.390
		Conservative	**8.302**	2.934	4.694	5.396	7.837	11.389	12.968	17.461
		Lifecycle (TDF)	**9.001**	2.900	4.773	5.401	8.447	12.893	14.901	21.477
High volatility	**Gaussian with time-varying parameter**	Balanced	**10.141**	2.088	4.218	5.340	9.305	15.728	18.755	26.882
		Conservative	**8.357**	2.777	4.494	5.365	7.981	11.351	13.008	18.757
		Lifecycle (TDF)	**9.175**	2.618	4.543	5.391	8.573	13.029	15.031	22.623
Low correlation	**Gaussian with time-varying parameter**	Balanced	**9.985**	2.529	4.195	5.240	9.062	15.379	18.705	27.732
		Conservative	**8.377**	2.972	4.637	5.430	7.943	11.477	12.982	20.110
		Lifecycle (TDF)	**9.150**	3.179	4.654	5.458	8.522	12.998	15.181	24.301
High correlation	**Gaussian with time-varying parameter**	Balanced	**9.990**	2.629	4.464	5.471	9.002	15.130	18.343	30.668
		Conservative	**8.286**	2.939	4.741	5.492	7.887	11.265	12.940	18.673
		Lifecycle (TDF)	**9.036**	3.365	4.678	5.541	8.411	12.796	14.883	22.876

Table 12: Mean and quantiles of RWRs for the three strategies according to the time-varying Gaussian copula under different regimes

effect of the occurrence of a crisis during the last 10 years of an investor's contribution on the terminal wealth. This analysis is related to what is generally known as *sequencing risk* and has provided valuable insights in retirement funds analysis, see e.g. Doran et al. (2012). We assume that a market crash or significant drop in equity prices occurs at some point over the last 10 years of contributions. In We consider three different scenarios: (i) we assume that the crisis happens at the beginning (i.e. in year one) of the 10 year period, (ii) we assume that the crisis happens in the middle (i.e. in year five or six) of the 10 year period, and, (iii) we assume that the crisis year happens at the end (i.e. in year 10) of the contribution period. In our simulation procedure, we set the returns for the market crash period equal to actually observed returns during the 2007-2008 global financial crisis.

In Table 13 we present the results. As expected, the presence of a market crash over the last 10 year of superannuation contributions does have a significant impact on the terminal wealth. Our results indicate that a crisis at the beginning of the 10 year period has the most negative effect on the terminal RWR. When a crisis occurs in the middle or the end there is an impact on the accumulated value to that point but the final wealth is still higher. Interestingly, although we only consider the last 10 years of contribution, the TDF strategy is only more suitable than the others if the crisis occurs in the final year. If the crisis occurs at the beginning of the 10 year period the defensive strategy is more suitable and if it occurs in the middle the balanced MySuper strategy with the highest share in growth assets yields the highest wealth outcomes.

It has been argued in the literature that having a crisis as an investor approaches the age of retirement can have catastrophic consequences. Although we do not argue the opposite, such a recommendation may be based mainly on the amount of money that is lost during the occurrence of a crisis. However, one should not ignore the potentially significantly higher performance of equity markets in comparison to more defensive assets over the period before the crisis. Also, since markets often recover quickly after a crisis period, a higher investment in growth assets also has the potential to achieve significant capital gains during such periods of recovery. Given our results, further research is required on the optimal timing of a possible switch from growth to more defensive assets. However, our results do not clearly indicate a superior performance of lifecycle or conservative strategies over the last 10 years, even when a substantial market crash takes place during this period.

Regime	MODEL	Strategy	MEAN	Quantile for RWR						
			RWR	**1%**	**5%**	**10%**	**50%**	**90%**	**95%**	**99%**
No crisis	**Block Bootstrap with equal weights block size n=6**	Balanced	**9.840**	1.998	4.182	5.285	9.102	14.685	17.580	26.705
		Conservative	**8.241**	2.934	4.701	5.518	7.925	11.068	12.559	17.326
		Lifecycle (TDF)	**8.977**	2.776	4.593	5.496	8.488	12.650	14.364	21.821
Crisis in the beginning	**Block Bootstrap with equal weights block size n=6**	Balanced	**7.660**	1.662	3.003	3.686	6.847	11.904	14.679	25.904
		Conservative	**7.929**	3.185	4.434	5.086	7.534	10.827	12.396	19.072
		Lifecycle (TDF)	**7.421**	2.430	3.639	4.245	6.874	10.773	12.799	20.645
Crisis in the middle	**Block Bootstrap with equal weights block size n=6**	Balanced	**7.994**	2.323	3.623	4.424	7.664	11.803	13.536	19.244
		Conservative	**7.743**	3.349	4.750	5.385	7.439	10.251	11.327	15.370
		Lifecycle (TDF)	**7.857**	2.841	4.342	5.109	7.555	10.694	12.091	17.183
Crisis in the end	**Block Bootstrap with equal weights block size n=6**	Balanced	**7.970**	1.458	3.199	4.203	7.449	11.769	14.041	20.843
		Conservative	**7.836**	2.781	4.534	5.213	7.562	10.417	11.815	16.834
		Lifecycle (TDF)	**8.262**	2.744	4.280	5.014	7.814	11.470	13.541	20.783

Table 13: Mean and quantiles of RWRs for the three strategies according to the time-varying Gaussian copula under different regimes of crisis

5.6. The impact of the endowment, income and contribution levels

Other very important factors impacting on the terminal wealth of a superannuation investor, we have not discussed so far, are the amount accumulated up to ten years before retirement as well as salary and the contribution levels. We finalize our analysis by considering different representative examples.

5.6.1. Initial balances

In Table 14, we consider the impact of an initial balance of \$200,000 and \$300,000 in comparison to an initial balance of \$250,00 for the exemplary superannuation investor. Note that here we only provide results for the block bootstrap simulation with equal weights and block size $n = 6$, however, results for other simulation techniques were quite similar and led to the same conclusion. We find that the effect of the initial balance on the terminal wealth seems to be almost linear: the quantiles and means presented for an initial endowment of \$250,00 are very close to the average of the RWR outcomes for an initial balance of of \$200,000 and \$300,000.

5.6.2. Salaries and contribution levels

In Table 15 we present the probability of reaching the ASFA comfortable lifestyle according to three different incomes and two contribution levels. Hereby, we assume an initial balance of the investor of \$250,000 and consider the three quartiles, i.e. the 25th, 50th and 75th percentile of the Australian income distribution. We examine results for a base scenario contribution level of 9.5% and also an increased contribution level of 12%. Our results indicate that contributing 12% instead of 9.5% has a very signifiacnt impact on the probability of achieving or exceeding the AFAS threshold. OUr findings illustrate that an increase in contributions from 9.5% to 12% almost has the same effect as going from one quartile of the income distribution to the next higher quartile. Thus, one recommendation from this exercise is that in order to increase the chances of reaching a terminal wealth equal or greater than the AFAS comfortable standard of living, investors with lower income and a standard contribution level should prefer the balanced MySuper strategy. For investors with higher income the strategy is not that relevant in order to get to this level, since the probability of achieving at least the AFAS threshold of comfortable lifestyle is greater than 90% for all strategies.

MODEL	**Strategy**	**Mean RWR**	**Quantile for RWR**						
			1%	**5%**	**10%**	**50%**	**90%**	**95%**	**99%**
Initial Balance $250,000	Balanced	**9.6308**	3.3949	4.4004	5.3297	8.9881	14.9016	17.0258	22.6976
Block bootstrap	Conservative	**8.3041**	4.7262	5.4712	5.9489	8.0821	10.9521	11.8340	13.9327
(block size n = 6)	Lifecycle (TDF)	**8.8513**	4.1927	5.1049	5.7699	8.5105	12.4031	13.8196	17.5419
Initial Balance $200,000	Balanced	**7.9829**	2.8841	3.6999	4.4628	7.4708	12.2801	14.0360	18.6788
Block bootstrap	Conservative	**6.8974**	3.9663	4.5765	4.9698	6.7103	9.0443	9.7993	11.4737
(block size n = 6)	Lifecycle (TDF)	**7.3435**	3.5216	4.2729	4.8227	7.0704	10.2271	11.3895	14.4319
Initial Balance $300,000	Balanced	**11.2788**	3.9188	5.1017	6.2132	10.4953	17.5402	19.9962	26.7163
Block bootstrap	Conservative	**9.7109**	5.4990	6.3702	6.9270	9.4536	12.8542	13.8682	16.4034
(block size n = 6)	Lifecycle (TDF)	**10.3590**	4.8414	5.9383	6.6976	9.9635	14.5737	16.2573	20.6519

Table 14: Mean and quantiles of RWRs for the three strategies for an initial balance of $200,000, $250,000, and $300,000.

Level of income/ contribution	Model	Strategy		
		Bal	Cons	TDF
Low income/9.5%	**Gaussian with time-varying parameter**	0.575	0.406	0.493
Low income/12%		0.771	0.727	0.760
Median income/9.5%		0.805	0.776	0.804
Median income/12%		0.913	0.931	0.927
High income/9.5%		0.941	0.955	0.955
High income/12%		0.969	0.976	0.978

Table 15: Probability of reaching or exceeding the ASFA comfortable standard of living threshold for different levels of salary and contribution according to the Gaussian copula model with time-varying dependence parameter.

6. Conclusions

The aim of this paper was to deepen the understanding of different factors impacting on the wealth of a superannuation investor at retirement. In order to simulate retirement wealth outcomes, this is one of the first studies to apply both parametric and non-parametric models for the performance of different superannuation strategies in the Australian context.

For the parametric approach, we examine different copula models in order to model the dependence structure between Australian equity and bond returns. Copulas offer great flexibility for modeling the relationship between different financial variables and also provides insights with respect to nonlinear dependence between the asset classes. Thus, we first investigate which copulas are most appropriate to model the dependence structure. Second, we deal with the question whether or not the dependence structure exhibits time-varying properties. The latter allows us to examine whether the relationship between the considered variables has changed over time and whether or not financial crises have had an influence on the dependence between these two assets.

For the non-parametric approach we apply different block and stationary bootstrap methods. These techniques also allow us to capture the possible dependence between the returns of the considered asset classes as well as the existing autocorrelation structure of the returns. We also implement a bootstrap approach that allows us to allocate higher weights to more recent observations.

Based on the conducted simulation study, we then examine different MySuper portfolio strategies allocating different weights to investments in equity and bonds. We consider two different Target Risk Funds (TRFs) and one Target Date Fund (TDF) with a deterministic glide path from a balanced to a more conservative exposure. IN particular we consider a portfolio with 70% growth assets (equity) and 30% conservative assets (bonds) as an example for a typical Mysuper *balanced product*; a portfolio with 30% growth assets (equity) and 70% conservative assets as an example for a typical Mysuper *conservative investment product* and a portfolio that linearly switches from 59% investment in equities to 37% investment in equities, while the remaining fraction is invested in bonds.

The following insights emerge from our analysis. First, for the considered time series, we apply the Clayton, Gumbel, Gaussian and Student t copula to investigate the dependence structure between Australian stocks and bonds for superannuation portfolios. To our best knowledge this is one of the first studies to apply this technique in superannuation portfolios. The Gaussian and Gumbel copulas are most appropriate, significantly outperforming both the Clayton and Student t copula with respect to a goodness-of-fit test for the distance between the estimated and empirical copula. Second, a significant change in the nature of the dependence structure is found between the two assets. This is detected by the correlation parameter of the Gaussian copula, with a steep decline since the year 2000. The parameter for the Gumbel copula is more volatile but also detects a high variation in the dependence. This confirms general results on asset returns from financial markets exhibiting different regimes of dependence during periods of different market conditions.

We also provide a risk adjusted analysis for wealth outcomes of superannuation investors by using the retirement wealth ratio (RWR). Our analysis not only allows us to report average expected terminal wealth outcomes for investors, but also illustrates how likely it is to achieve or exceed defined threshold levels such as e.g. the lump sum required to support a comfortable lifestyle as suggested by the Association of Superannuation Funds of Australia (ASFA, 2014).

In a nutshell, we find that the use of different strategies only becomes very relevant when dealing with high RWRs, that is with high expectations of wealth. Interestingly, considering the last 10 years only, a balanced investment strategy with a higher share in equities than defensive assets, only provides a marginally lower risk to end up with very bad terminal wealth outcomes at retirement in comparison to a conservative or lifecycle strategy. On the other hand, the balanced strategy provides significantly higher upside potential, such that an investor who is mainly interested in maximizing the probability of having a relatively high RWR should prefer growth assets. On the other hand, very risk averse investors might still prefer a conservative or lifecycle strategy in order to increase the probability of exceeding at least a minimum retirement wealth ratio of 5.

We also find that if returns of stocks and bonds will behave similar to what could be observed over the last two decades, there is a clear need to lower expectations for terminal wealth outcomes at retirement. Thus, simulations using a historical sample period with return observations dating back to the 1970s where, for example, bond returns were substantially higher than in recent periods, might overestimate actual wealth outcomes for superannuation investors.

Interestingly, the effect of starting the last 10 years with a market regime of extreme volatility or correlation has only minor effects on the wealth outcome at retirement. On the other hand, a market crash similar to the one of the 2007-2008 GFC has a very significant impact on the outcome for all three strategies. This impact is higher when the crisis occurs in the beginning of the 10 year period, as we need to consider not only the money lost but its future profits. Interestingly, our results illustrate that even assuming that there will be a crash in equity markets during the last 10 years, it is not necessarily the optiml decision to choose a conservative or lifecycle strategy over a balanced MySuper strategy.

The effect of the initial accumulated wealth is close to linear on the terminal wealth. We also find that increasing the level of contribution has a huge impact on the terminal wealth, particularly for investors with low to medium income. We recommend that investors with low income who want to have a reasonable chance of achieving a comfortable standard of living should rather invest in balanced or growth strategies with higher investments in equity.

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