Coverage Analysis of HetNets with Base Station Cooperation and Interference Cancellation

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2014 HetNet Workshop

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Work supported by:
U.S. NSF (CNS 1014932 and CCF 1216407)

Coverage Analysis of HetNets with Base Station Cooperation and Interference Cancellation: Why the Waist-to-Hip Ratio is Essential

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Contents

Coverage

Small cells

A small cell.

Cell-ular user

A **cell** phone.

No noise

This is a completely noise-less talk.

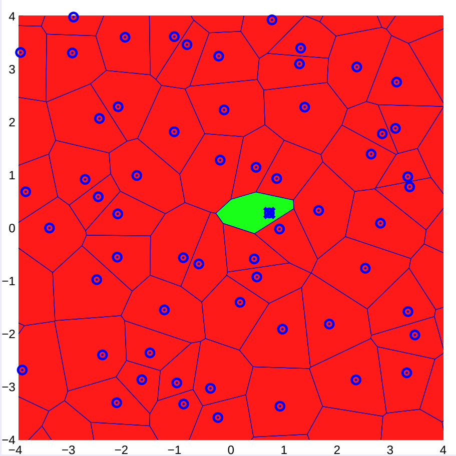
Menu

Overview

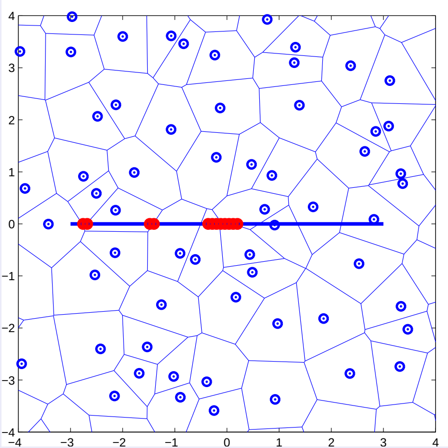
- Bird's view of cellular networks
- The HIP model and its properties
- Comparing SIR distributions: The crucial role of the mean interference-to-signal ratio (MISR)
- MISR gain due to BS silencing and cooperation
- DUI: Diversity under interference
- General BS cooperation
- Back to modeling: Inter- and intra-tier dependence
- Conclusions

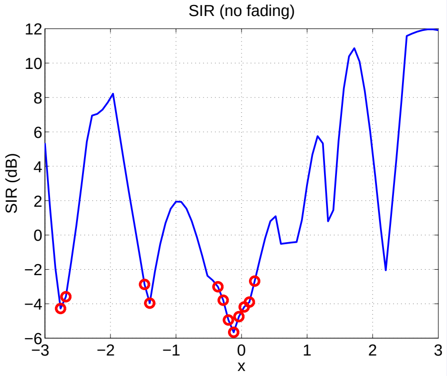
Big picture

Frequency reuse 1: A single friend, many foes

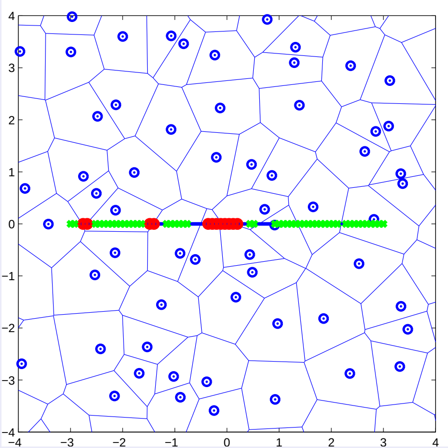


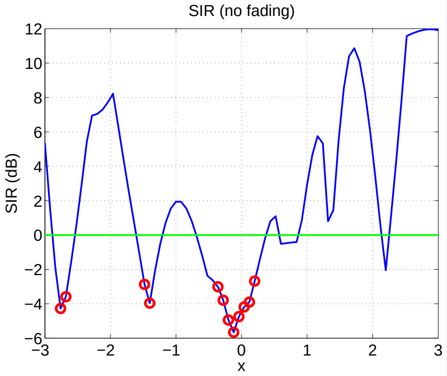
A walk through a single-tier cellular network





Coverage at 0 dB





SIR distribution

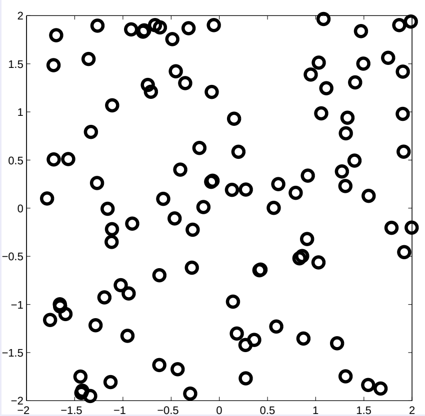
For ergodic models, the fraction of the curve that is above the threshold θ is the ccdf of the SIR at θ:

$$p_s(\theta) \triangleq \bar{F}_{SIR}(\theta) \triangleq \mathbb{P}(SIR > \theta)$$

It is the fraction of the users with SIR $> \theta$ if users are uniformly distributed.

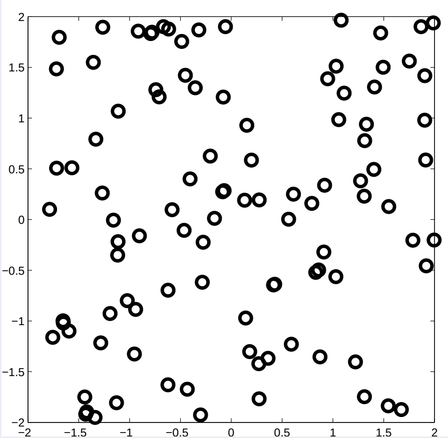
The HIP baseline model for HetNets

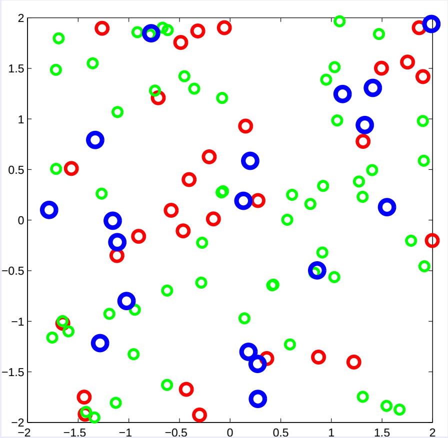
The HIP (homogeneous independent Poisson) model



Start with a homogeneous Poisson point process (PPP). Here $\lambda = 6$.
Then randomly color them to assign them to the different tiers.

The HIP (homogeneous independent Poisson) model





Randomly assign BS to each tier according to the relative densities. Here $\lambda_i = 1, 2, 3$. Assign power levels P_i to each tier.
This model is doubly independent and thus highly tractable.

Basic result for downlink

Assumptions:

- A user connects to the BS that is strongest on average, while all others interfere.
- Homogeneous path loss law $\ell(r) = r^{-\alpha}$ and Rayleigh fading.

Result for $\alpha = 4$:

$$p_s(\theta) = \mathbb{P}(\mathrm{SIR} > \theta) = \bar{F}_{\mathrm{SIR}}(\theta) = \frac{1}{1 + \sqrt{\theta} \arctan \sqrt{\theta}}.$$

Remarkably, this is independent of the number of tiers, their densities, and their power levels.
So as far as the SIR is concerned, we can replace the multi-tier HIP model by an equivalent single-tier model.
(For bounded path loss laws, this does not hold.)

Properties of the HIP model

For the unbounded path loss law:

- $\mathbb{E}(S) = \infty$ and $\mathbb{E}(\mathrm{SIR}) = \infty$ due to the proximity of the strongest BS.
- $\mathbb{E}(I) = \infty$ for $\alpha \geq 4$ due to the proximity of the strongest interferer.

The first two properties are not restricted to the Poisson model.

Remarks

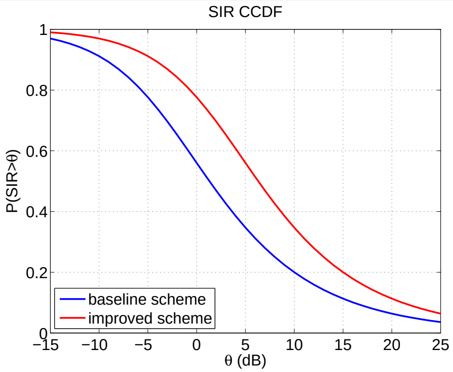
Per-user capacity improves with smaller cells, but coverage does not.
(Unless the interference benefit from inactive BSs kicks in.)

Question: How to boost coverage?
$\Rightarrow$ Non-Poisson deployment
$\Rightarrow$ BS silencing
$\Rightarrow$ BS cooperation

How to quantify the improvement in the SIR distribution?

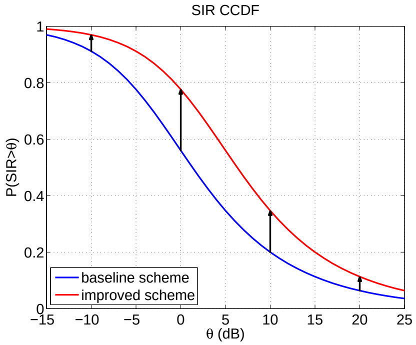
Comparing SIR distributions

Two distributions



How to quantify the improvement?

The standard comparison: vertical



At -10 dB, the gap is 0.058. Or 6.4%.

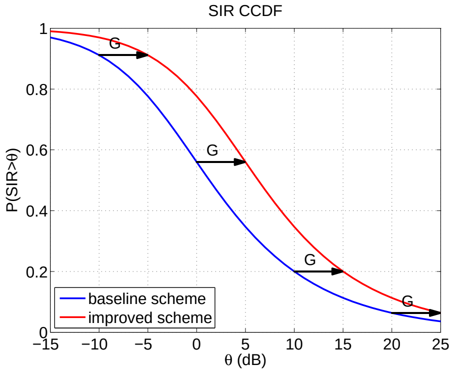
At 0 dB, the gap is 0.22. Or 39%.

At 10 dB, the gap is 0.15. Or 73%.

At 20 dB, the gap is 0.05. Or 78%.

Or use the gain in $\mathbb{P}(\mathrm{SIR} \leq \theta)$?

A better choice: horizontal



Use the horizontal gap instead.

This SIR gain is nearly constant over θ in many cases.

If the improvement is due to better BS deployment, it is the deployment gain.

$$p_s = \mathbb{P}(\mathrm{SIR} > \theta) \quad \Rightarrow \quad p_s = \mathbb{P}(\mathrm{SIR} > \theta/G).$$

Can we quantify this gain?

Horizontal gap at probability p

The horizontal gap between two SIR ccdfs is

$$G(p) \triangleq \frac{\bar{F}^{-1}_{\mathrm{SIR}_2}(p)}{\bar{F}^{-1}_{\mathrm{SIR}_1}(p)}, \quad p \in (0,1),$$

where $\bar{F}^{-1}_{\mathrm{SIR}}$ is the inverse of the ccdf of the SIR, and p is the target success probability.

We also define the asymptotic gain (whenever the limit exists) as

$$G \triangleq G(1) = \lim_{p\to 1} G(p).$$

Relevance

We will show that

- G is relatively easy to determine.
- $G(p) \approx G$ for all practical p.

The ISR

Definition ($\mathrm{I\bar{S}R}$)

The interference-to-average-signal ratio is

$$\mathrm{I\bar{S}R} \triangleq \frac{I}{\mathbb{E}_h(S)},$$

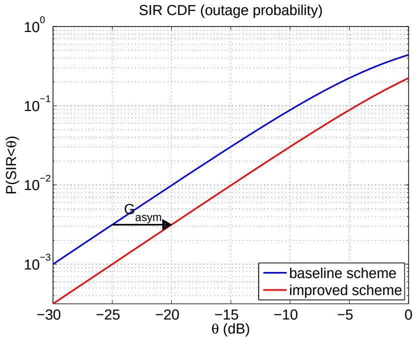
where $\mathbb{E}_h(S)$ is the desired signal power averaged over the fading.

Comments

- The $\mathrm{I\bar{S}R}$ is a random variable due to the random positions of BSs and users. Its mean MISR is a function of the network geometry only.
- If the desired signal comes from a single BS at distance R, $\mathrm{I\bar{S}R} = IR^\alpha$.
- If the interferers are located at distances R_k,

$$\mathrm{MISR} \triangleq \mathbb{E}(\mathrm{I\bar{S}R}) = \mathbb{E}\left(R^\alpha \sum h_k R_k^{-\alpha}\right) = \sum \mathbb{E}\left(\frac{R}{R_k}\right)^\alpha.$$

Relevance of the $\mathrm{I\bar{S}R}$



$$p_{\mathrm{out}} = \mathbb{P}(hR^{-\alpha} < \theta I) = \mathbb{P}(h < \theta\,\mathrm{I\bar{S}R})$$

For exponential h and $\theta \to 0$,

$$\mathbb{P}(h < \theta\,\mathrm{I\bar{S}R} \mid \mathrm{I\bar{S}R}) \sim \theta\,\mathrm{I\bar{S}R},$$

$$\text{thus} \quad \mathbb{P}(h < \theta\,\mathrm{I\bar{S}R}) \sim \theta\,\mathbb{E}(\mathrm{I\bar{S}R}).$$

So the asymptotic gain is

$$G \triangleq \mathbb{E}(\mathrm{I\bar{S}R}_1)/\mathbb{E}(\mathrm{I\bar{S}R}_2)\ .$$

So the gain is the ratio of the two MISRs.

How accurate is the asymptotic gain for non-vanishing θ?

The $\bar{\mathrm{IS}}\mathrm{R}$ for the HIP model

For the (single-tier) HIP model, we need to calculate the MISR

$$\mathbb{E}(\mathrm{I\bar{S}R}) = \mathbb{E}\left(R_1^{\alpha}\sum_{k=2}^{\infty} R_k^{-\alpha}\right) = \sum_{k=2}^{\infty}\mathbb{E}\left(\frac{R_1}{R_k}\right)^{\alpha},$$

where R_k is the distance to the k-th nearest BS.
The distribution of $\nu_k = R_1/R_k$ is

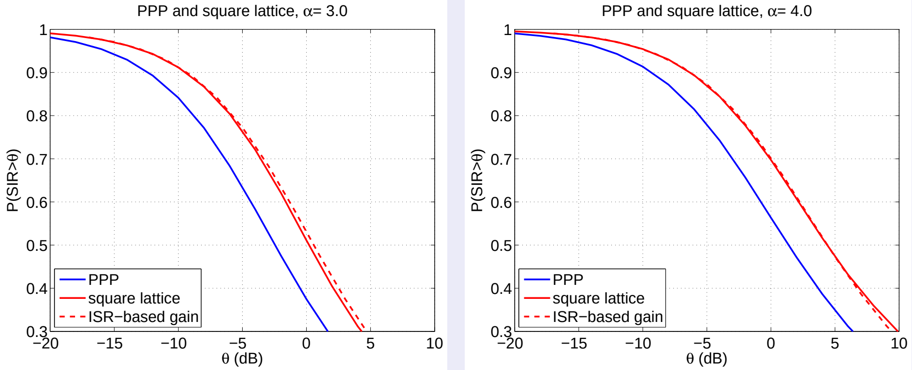
$$F_{\nu_k}(x) = 1-(1-x^2)^{k-1}, \quad x \in [0,1].$$

Summing up the α-th moments $\mathbb{E}(\nu_k^{\alpha})$, we obtain (remarkably)

$$\mathbb{E}(\mathrm{I\bar{S}R}) = \frac{2}{\alpha-2}.$$

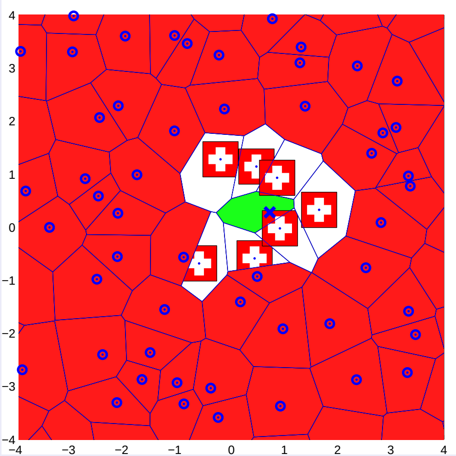
This is the baseline $\mathbb{E}(\mathrm{I\bar{S}R})$ relative to which we measure the gain.
For $\alpha = 4$, it is 1. Hence $p_{\mathrm{out}}(\theta) = F_{\mathrm{SIR}}(\theta) \sim \theta$, $\theta \to 0$.

Deployment gain



- For the square lattice, the gap (deployment gain) is quite exactly 3 dB—irrespective of α! For $\alpha = 4$, $p_s^{\mathrm{sq}} = (1 + \sqrt{\theta/2}\arctan\sqrt{\theta/2})^{-1}$.
- For the triangular lattice, it is 3.4 dB. This is the maximum achievable.

BS silencing: neutralize nearby foes



Gain due to BS silencing (or interference cancellation) for HIP model

Let $\mathrm{I\bar{S}R}^{(!n)}$ be the $\mathrm{I\bar{S}R}$ obtained when the n strongest (on average) interferers are silenced.

For HIP,

$$\mathbb{E}(\mathrm{I\bar{S}R}^{(!n)}) = \frac{2\Gamma(1+\alpha/2)}{\alpha-2}\,\frac{\Gamma(n+2)}{\Gamma(n+1+\alpha/2)}.$$

For $\alpha = 4$, in particular,

$$\mathbb{E}(\mathrm{I\bar{S}R}^{(!n)}) = \frac{2}{n+2}.$$

So the gain from silencing n BSs is simply

$$G_{\text{silence}} = 1 + \frac{n}{2}.$$

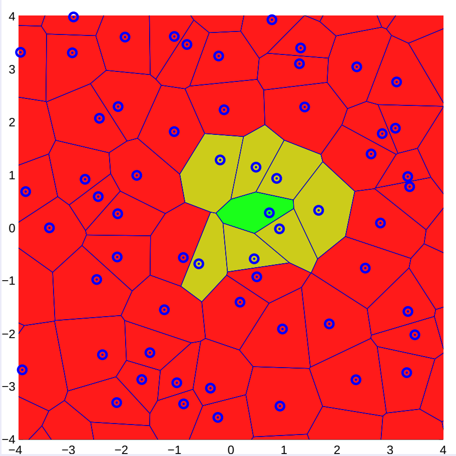
Remarks on interference cancellation

- The gain from successive canceling interferers is highest if transmitters are clustered.
- For a non-homogeneous Poisson model with intensity function $\lambda(x) = a\|x\|^b$, $b \in (-2, \alpha - 2)$, the closer b to -2, the better.
- If small-cell BSs are clustered due to high user density or if closed-access femtocells exist, SIC is promising.
- Probability of decoding k-th strongest transmitter if $k-1$ have already been decoded, for arbitrary fading:

$$\text{For } \theta \geq 1 \ : \quad P_k = \frac{1}{\theta^{k\beta}\Gamma(1+k\beta)(\Gamma(1-\beta))^k},$$

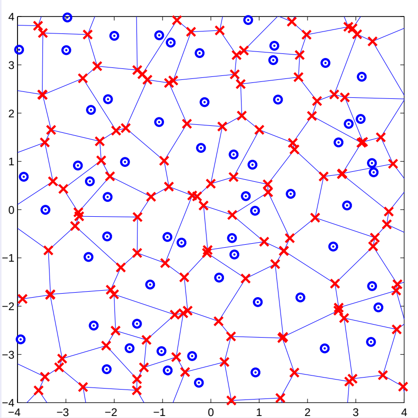
where $\beta = (2+b)/\alpha \in (0,1)$.

BS cooperation: turn nearby foes into friends



Cooperation for worst-case users

SIR at Voronoi vertices with cooperation



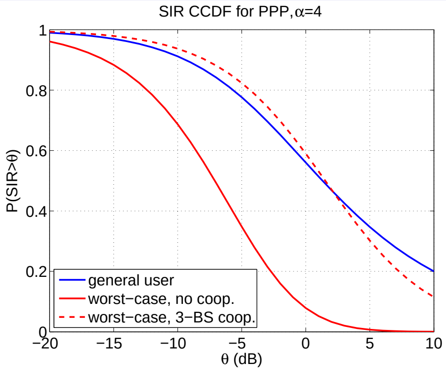
- At these locations (×), the user is far away from any BS, and there are two interfering BS at the same distance.
- In the Poisson model,

$$\bar{F}^{\times}_{\text{SIR}}(\theta) = \frac{\bar{F}^{2}_{\text{SIR}}(\theta)}{(1+\theta)^2}.$$

- With BS cooperation from the 3 equidistant BSs, for $\alpha = 4$,

$$\bar{F}^{\times,\text{coop}}_{\text{SIR}}(\theta) = \bar{F}^{2}_{\text{SIR}}(\theta/3) = \left(1 + \sqrt{\theta/3}\,\arctan(\sqrt{\theta/3})\right)^{-2}.$$

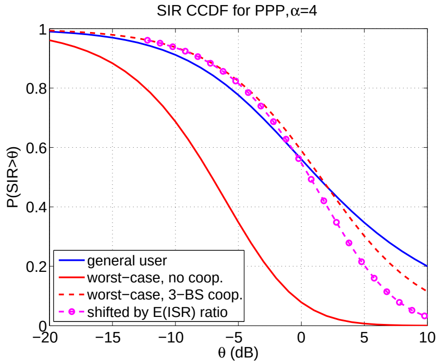
SIR at Voronoi vertices with cooperation



- Without cooperation, $\mathbb{E}(\mathrm{I\bar{S}R}) = 4$ (for $\alpha = 4$).
- With 3-BS cooperation, the SIR performance is slightly better for the worst-case users than the general users, and $\mathbb{E}(\mathrm{I\bar{S}R}) = 2/3$.
- So the gain from 3-BS cooperation is 6, or 7.8 dB.

Can we approximate the SIR distribution using the ratio of the two MISRs?

Using the ISR approximation



For worst-case users with $n \in \{1,2,3\}$ BSs cooperating,

$$\mathbb{E}(\bar{\mathrm{ISR}}) = \frac{4 + (3-n)(\alpha-2)}{n(\alpha-2)}.$$

So for $n = 3$, the ratio of the two MISRs is

$$G_{\mathrm{coop}} = \frac{\mathrm{MISR}}{\mathrm{MISR}_{\mathrm{coop}}} = 3 + \frac{3}{2}(\alpha-2).$$

DUI: Diversity under interference

Decreasing the variability in the SIR

The shift along the θ axis preserves the variability in the SIR.
The variability can be decreased by increasing the diversity.

Definition (DUI)

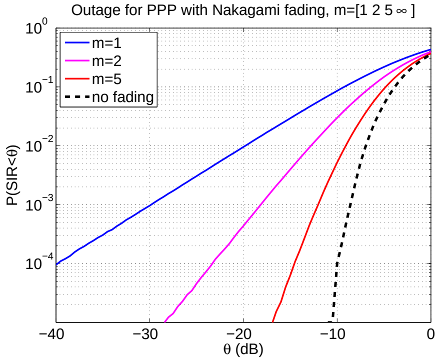
The diversity under interference is defined as

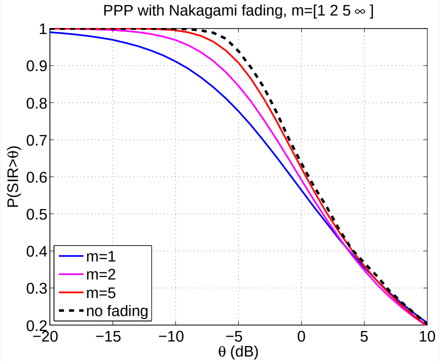
$$d \triangleq \lim_{\theta \to 0} \frac{\log p_{\mathrm{out}}(\theta)}{\log \theta}.$$

Hence

$$p_{\mathrm{out}}(\theta) = F_{\mathrm{SIR}}(\theta) = \Theta(\theta^d).$$

Diversity for HIP model





For a PPP with Nagakami-m fading, $d = m$.

SIR gain with diversity

If the fading distribution satisfies $F_h(x) \sim ax^m$, $x \to 0$, (Nakagami fading)

$$p_{\text{out}}(\theta) \sim a\theta^m \mathbb{E}(\mathrm{I\bar{S}R}^m).$$

The diversity order is m—if the m-th moment of the $\mathrm{I\bar{S}R}$ is finite.

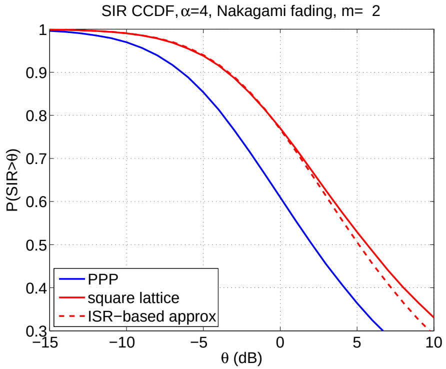
The asymptotic gain is

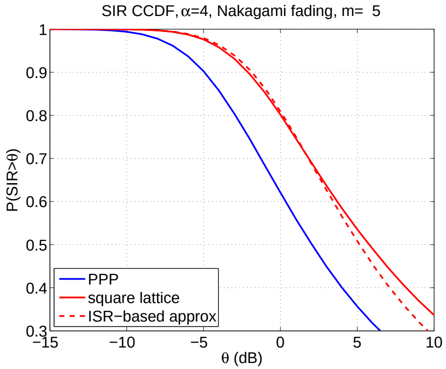
$$G^{(m)} = \left(\frac{\mathbb{E}(\mathrm{I\bar{S}R}_1^m)}{\mathbb{E}(\mathrm{I\bar{S}R}_2^m)} \right)^{1/m} \approx G^{(1)}.$$

For the PPP, all moments of the $\mathrm{I\bar{S}R}^m$ are finite.

For a large class of mixing motion-invariant point process models, the moments $\mathbb{E}(\mathrm{I\bar{S}R}^m)$ exist, and all outage curves have the same slope. The exact necessary conditions have not been established.

Deployment gain for Nakagami fading





General BS sharing

Fluid sharing of BS resources

Take a non-increasing function $f\colon \mathbb{R}^+ \mapsto \mathbb{R}^+$.
Let (r_1, r_2, r_3) be the triplet of distances to the 3 nearest BSs.

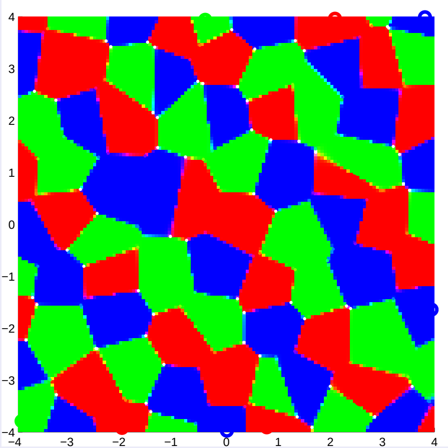
$$s = f(r_1) + f(r_2) + f(r_3)$$

Allocate resources according to $f(r_i)/s$ from BS i.

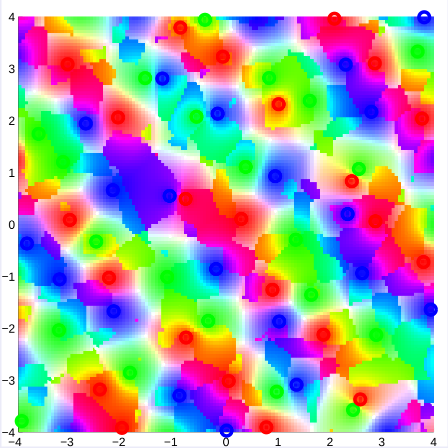
Example:

- $f(r) = r^{-\alpha}$. For $\alpha \to \infty$, this is the non-sharing (hard) scheme.
- $f(r) \equiv 1$: BS sharing with equal powers.

Towards amorphous networks

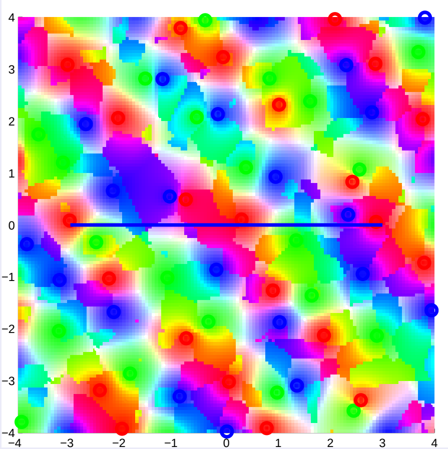


No BS sharing

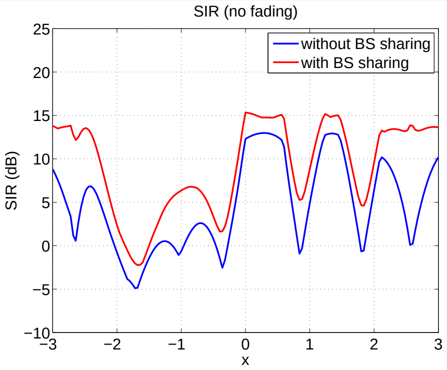


BS sharing with $\alpha = 4$

"SIR walk"



BS sharing with $\alpha = 4$



SIR improvement at equal total power consumption

This BS sharing framework can be reversed for the uplink in a natural way. There is hope that such a fluid BS sharing model is tractable.

Back to modeling

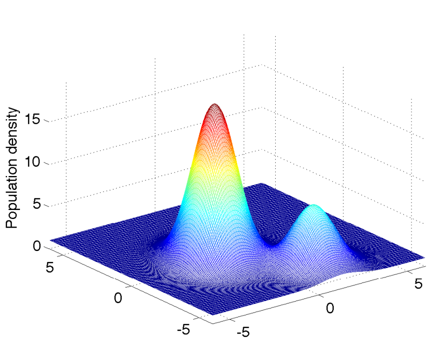
Introducing dependencies

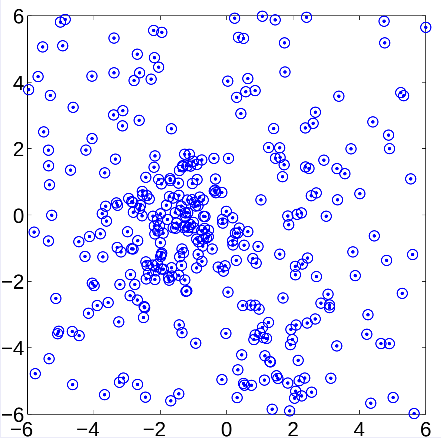
- BS of one tier are not placed completely independently (intra-tier dependence).
- BS of different tiers are not placed independently, either (inter-tier dependence).

Intra-tier dependence

The HIP model is conservative since it places BSs arbitrarily close to each other.
In actual deployments, it is unlikely to have two BSs very close, so the BSs form a soft-core or hard-core process. In other words, the BSs process is repulsive.
Repulsion can be quantified using the pair correlation function $g(r)$. For the PPP, $g(r) \equiv 1$. For a repulsive process, $g(r) < 1$.

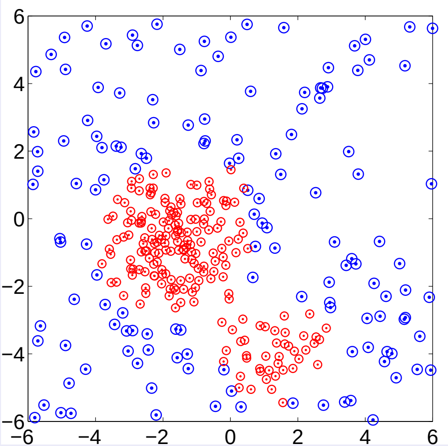
Adjusting to population density



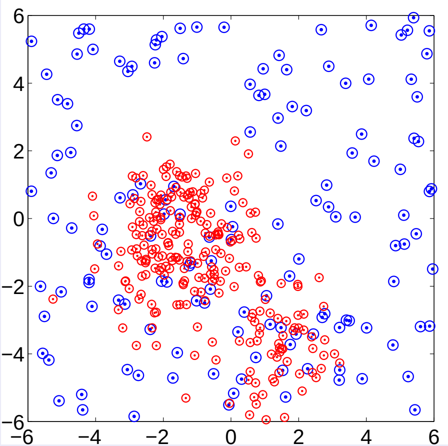


A single tier with density adjusted according to the population density to keep the average number of (active) users per cell constant.

Two-tier models



Small cells for urban regions

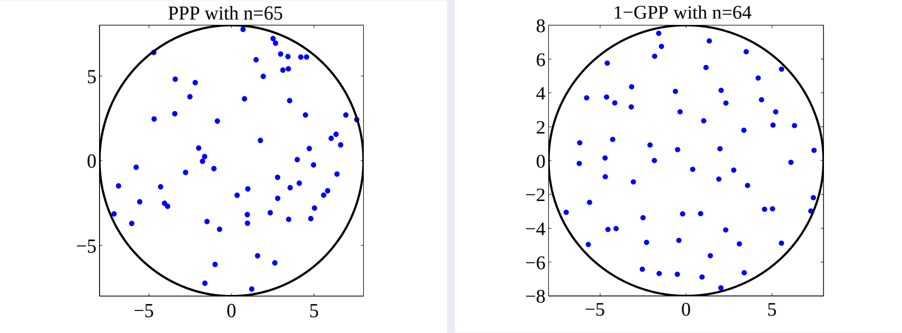


Overlay with small cells

Either way, there is intra- and inter-tier dependence.

The Ginibre model

Realizations of PPP and the Ginibre point process (GPP) on $b(o,8)$



The GPP exhibits repulsion—just as BSs in a cellular network.
Its pair correlation function is $g(r) = 1 - e^{-r^2}$.

The Ginibre point process

The GPP is a motion-invariant determinantal point process.

Remarkable property: If $\Phi = \{x_1, x_2, \ldots\} \subset \mathbb{R}^2$ is a GPP, then

$$\{\|x_1\|^2, \|x_2\|^2, \ldots\} \stackrel{\mathrm{d}}{=} \{y_1, y_2, \ldots\}$$

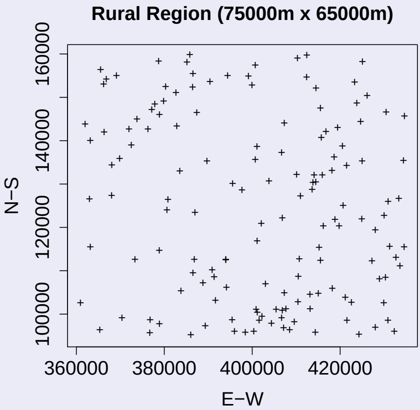
where (y_k) are independent gamma distributed random variables with pdf

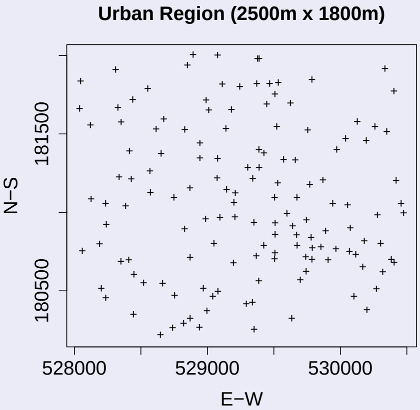
$$f_{y_k}(x) = \frac{x^{k-1} e^{-x}}{\Gamma(k)}; \qquad \mathbb{E}(y_k) = k.$$

- Removing y_1 from the process yields the Palm measure.
- The intensity is $1/\pi$ but can be adjusted by scaling.
- The GPP can be made less repulsive by independently deleting points with probability $1 - \beta$ and re-scaling. This β-GPP approaches the PPP in the limit as $\beta \to 0$.

The Ginibre point process in action

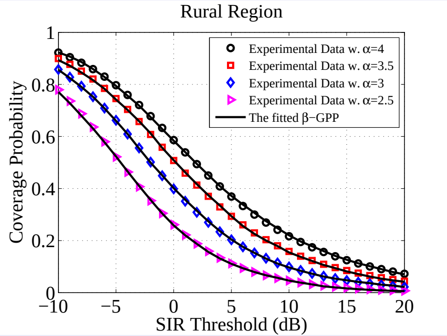
We would like to model these two deployments:





The Ginibre point process in action

SIR distributions for different path loss exponents:



For the rural region, $\beta = 0.2$. For the urban region, $\beta = 0.9$.

Conclusions

- The gain of a deployment/architecture/scheme is best measured as the horizontal gap relatively to a baseline (e.g., the HIP) model.
- The MISR $\mathbb{E}(\mathrm{I\bar{S}R})$ is easy to obtain by simulation, since it does not depend on the fading. The ISR-based approximation is very accurate for $p_{\mathrm{s}} > 3/4$, and the gains are quite insensitive to the path loss exponent and the fading statistics.
- The DUI "compresses" the SIR distribution. Care is needed due to the correlation in the interference across time and space. The existence of a diversity gain is coupled with the moments of the $\mathrm{I\bar{S}R}$.
- General BS sharing is a promising framework to analyze amorphous networks.
- Future work should also include models with intra- and inter-tier dependence. The Ginibre point process is promising as a repulsive model due to its tractability.

Key Take-away: What matters is the

WAIST-HIP Ratio

$$\frac{\mathrm{MISR}_{\mathrm{HIP}}}{\mathrm{MISR}_{\mathrm{WAIST}}}$$

WAIST: Wireless Advanced Interference Suppression Technique

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See `http://www.nd.edu/~mhaenggi/pubs` for our publications.