HetNets: what tools for analysis?

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Motivation

Seven Ways that HetNets are a Cellular Paradigm Shift, by J. Andrews, IEEE Communications Magazine, March 2013

Aspect	Traditional Cellular	HetNet
Performance Metric	Outage/coverage probability distribution (in terms of SINR) or spectral efficiency (bps/Hz)	Outage/coverage probability distribution (in terms of *rate*) or area spectral efficiency (bps/Hz/m^2)
Topology	BSs spaced out, have distinct coverage areas. Hexagonal grid is an ubiquitous model for BS locations.	Nested cells (pico/femto) inside macrocells. BSs are placed opportunistically and their locations are better modeled as a random process.
Cell Association	Usually connect to strongest BS, or perhaps two strongest during soft handover	Connect to BS(s) able to provide the highest data rate, rather than signal strength. Use biasing for small BSs.
Downlink vs. Uplink	Downlink and uplink to a given BS have approximately the same SINR. The best DL BS is usually the best in UL too.	Downlink and uplink can have very different SINRs; should not necessarily use the same BS in each direction
Mobility	Handoff to a stronger BS when entering its coverage area, involves signaling over wired core network	Handoffs and dropped calls may be too frequent if use small cells when highly mobile, overhead a major concern.
Backhaul	BSs have heavy-duty wired backhaul, are connected into the core network. BS to MS connection is the bottleneck.	BSs often will not have high speed wired connections. BS to core network (backhaul) link is often the bottle-neck in terms of performance and cost.
Interference Management	Employ (fractional) frequency reuse and/or simply tolerate very poor cell edge rates. All BSs are available for connec-tion, i.e. "open access"	Manage closed access interference through resource allocation; users may be "in" one cell while communicat-ing with a different BS; interference management hard due to irregular backhaul and sheer number of BSs

Outline

- Part I: MWBM

  MWBM is a special LP for which efficient polynomial-time algorithms exist, for example, the Hungarian algorithm.
  Useful to approximate at high-SNR “MIMO terms”.
  Ex: gDoF of Gaussian broadcast networks with relays and user scheduling decisions [arXiv:1304.5790, M. Cardone *et al*]

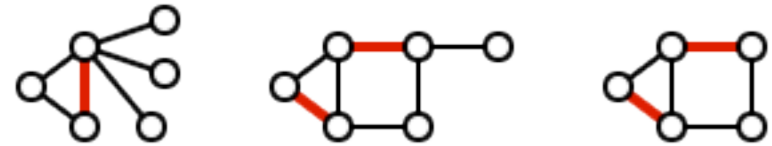
- Part II: MixIn + TIN

  To reduce control-plane overhead, design PHY robust to asynchronism and lack of user coordination.
  Ex: “Mixed inputs'” and “treat interference as noise”: optimal to within a log(log(SNR)) gap [arXiv:1401.5536 A. Dytso *et al*]

Part I

Maximum Weight Bipartite Matching

Definitions

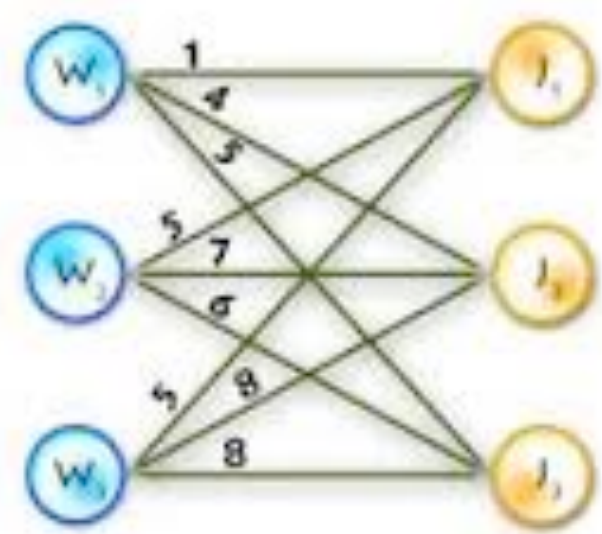


- A **matching**, or independent edge set, in a graph G=(V,E) is a set of edges without common vertices.
- In a weighted bipartite graph, each edge has an associated value. A **maximum weighted bipartite matching** (MWBM), or assignment problem, is a matching where the sum of the values of the edges in the matching have a maximal value.
- The Hungarian algorithm solves the assignment problem in O(V^2 E); it uses a modified shortest path search in the augmenting path algorithm.

Example

Each link (from a Tx antenna to a Rx antenna) has a `weight' given by its power expressed in dB

$$\begin{pmatrix} 1 & 4 & 5 \\ 5 & 7 & 6 \\ 5 & 8 & 8 \end{pmatrix} \Leftrightarrow$$



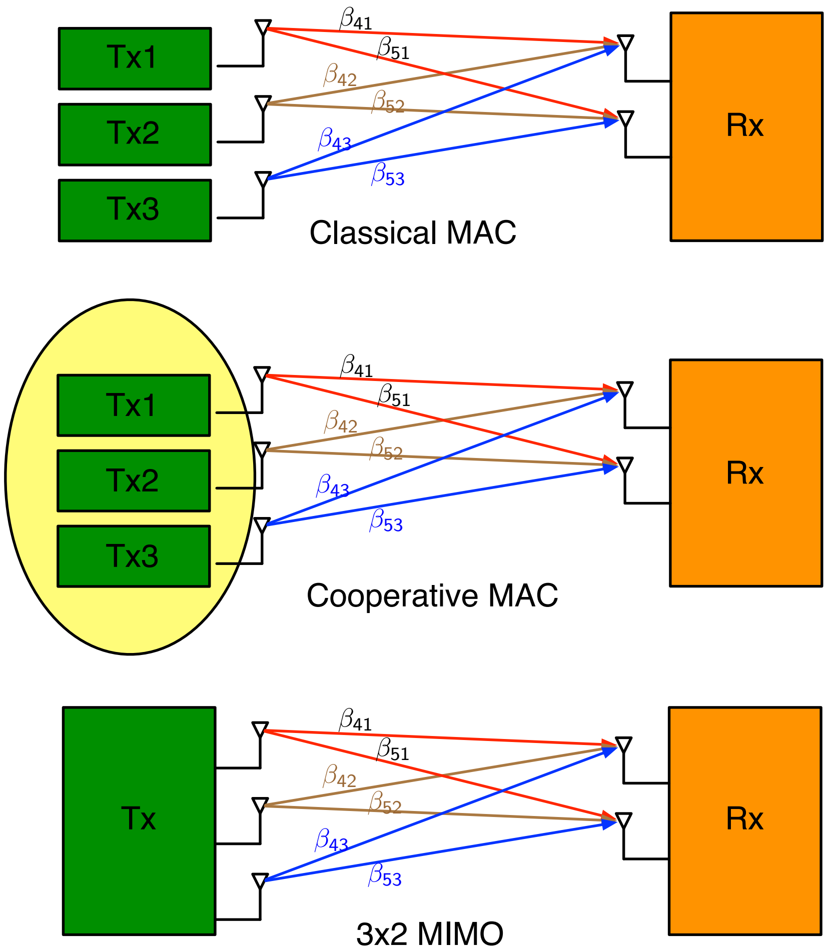
MIMO-type Setting

$$Y = \mathbf{H}X + Z \in \mathbb{C}^{n_R \times 1}, \ \mathbf{H} \in \mathbb{C}^{n_R \times n_T},$$
$$Z \sim N(0, \mathbf{I}_{n_R}) \text{ independent of } X,$$

$$C_1 : X \in \mathbb{C}^{n_T \times 1} : \mathbb{E}[|X_i|^2] \leq 1, \ i \in [1:n_T], \ \text{independent},$$
$$C_1 : X \in \mathbb{C}^{n_T \times 1} : \mathbb{E}[|X_i|^2] \leq 1, \ i \in [1:n_T],$$
$$C_2 : X \in \mathbb{C}^{n_T \times 1} : \sum_{i \in [1:n_T]} \mathbb{E}[|X_i|^2] \leq n_T,$$

$$I(X;Y) := \log \left| \mathbf{I}_{n_R} + \mathbf{H}\Sigma_x \mathbf{H}^H \right|$$
$$\Sigma_x := \mathbb{E}[XX^H] \succeq 0_{n_T}$$

MIMO-type Setting

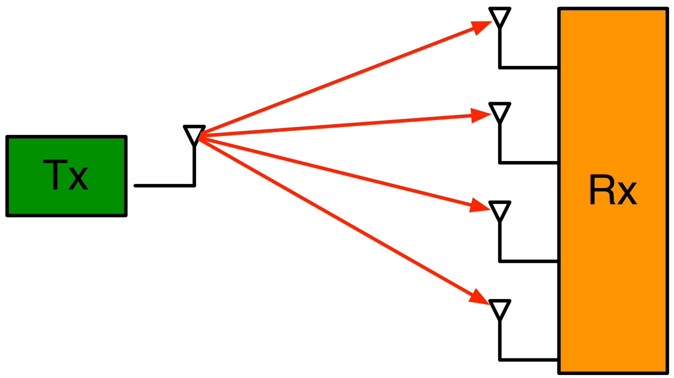


$$
\begin{aligned}
C_1 &= \log \left| \mathbf{I}_{n_R} + \mathbf{H}\,\mathbf{H}^H \right| \\
&\le C_2 \le C_3 \\
&\le \log \left| \mathbf{I}_{n_R} + \mathbf{H}\,\mathbf{H}^H \; n_T \right|
\end{aligned}
$$

$$
\begin{aligned}
C_3 - C_1 &\le \mathrm{rank}[\mathbf{H}] \; \log(n_T) \\
&\le \min(n_T, n_R) \log(n_T)
\end{aligned}
$$

Up to a constant, Tx power constraint does not matter

Example: SIMO (or SISO BC)



$$[\mathbf{H}]_{ij} := \sqrt{\mathsf{SNR}^{\beta_{ij}}}\exp(j\theta_{ij})$$

$$\mathbf{H} := [h_1, \dots, h_{n_R}]^T \longleftrightarrow \mathbf{B} := [\beta_1, \dots, \beta_{n_R}]^T$$

$$\log\left(1 + \mathsf{SNR}^{\max_i\{\beta_i\}}\right)$$

$$\leq C_1 = C_2 = C_3 = \log\left(1 + \sum_i \mathsf{SNR}^{\beta_i}\right)$$

$$\leq \log\left(1 + \mathsf{SNR}^{\max_i\{\beta_i\}}\right) + \log(n_R).$$

Up to a constant, Rx processing does not matter

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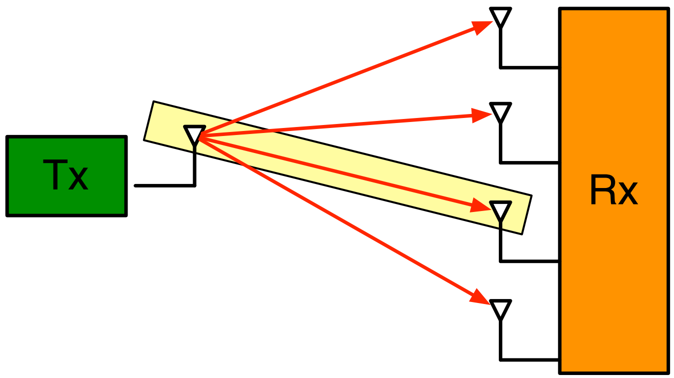

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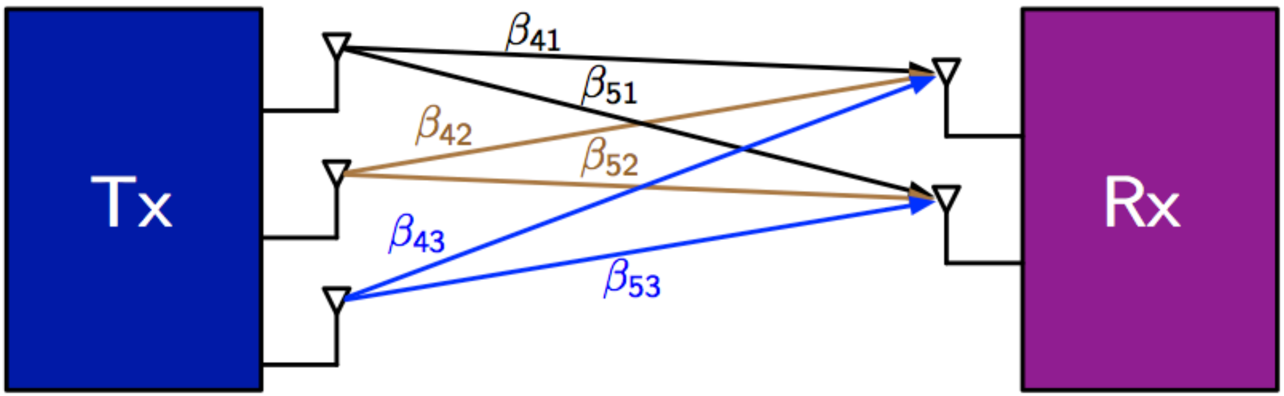
Up to a constant, Rx processing does not matter

$$\log \left| \mathbf{I} + \mathbf{H}\mathbf{H}^H \right| \approx \mathrm{MWBM}(\mathbf{B}) \log \left(1 + \mathsf{SNR}\right)$$



$$\mathsf{d}_1 = \beta_{41} + \beta_{52} \qquad \mathsf{d}_4 = \beta_{51} + \beta_{43}$$
$$\mathsf{d}_2 = \beta_{41} + \beta_{53} \qquad \mathsf{d}_5 = \beta_{42} + \beta_{53}$$
$$\mathsf{d}_3 = \beta_{51} + \beta_{42} \qquad \mathsf{d}_6 = \beta_{52} + \beta_{43}$$

$$\mathbf{d} = \max \{\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3, \mathbf{d}_4, \mathbf{d}_5, \mathbf{d}_6\}$$

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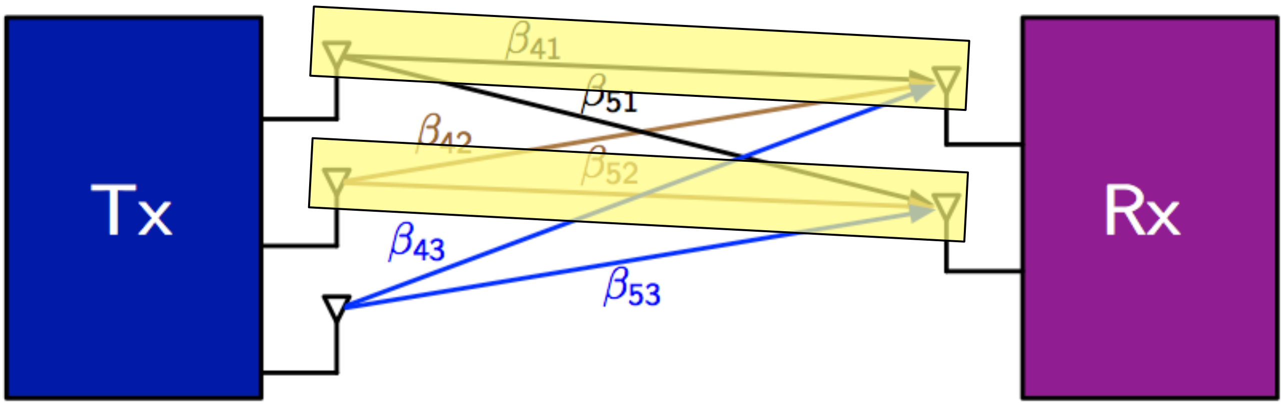

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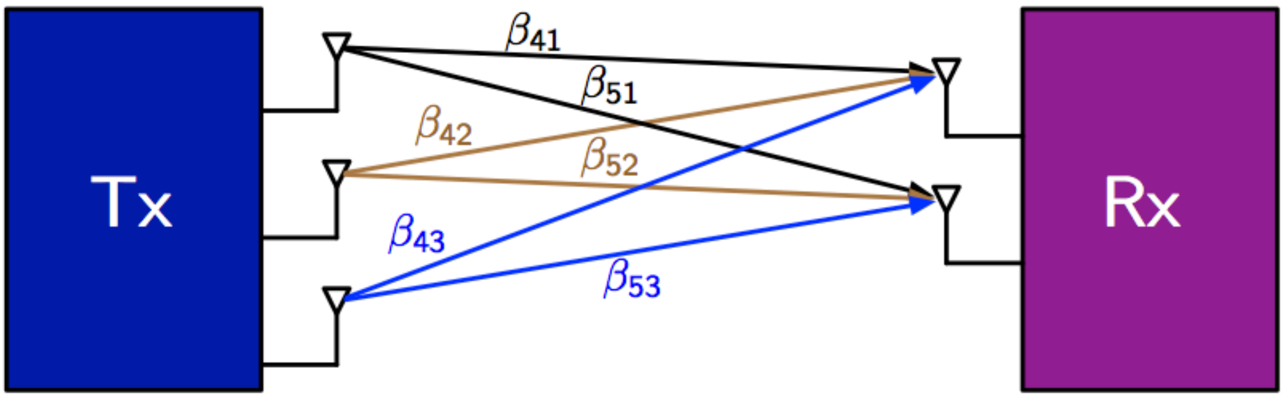

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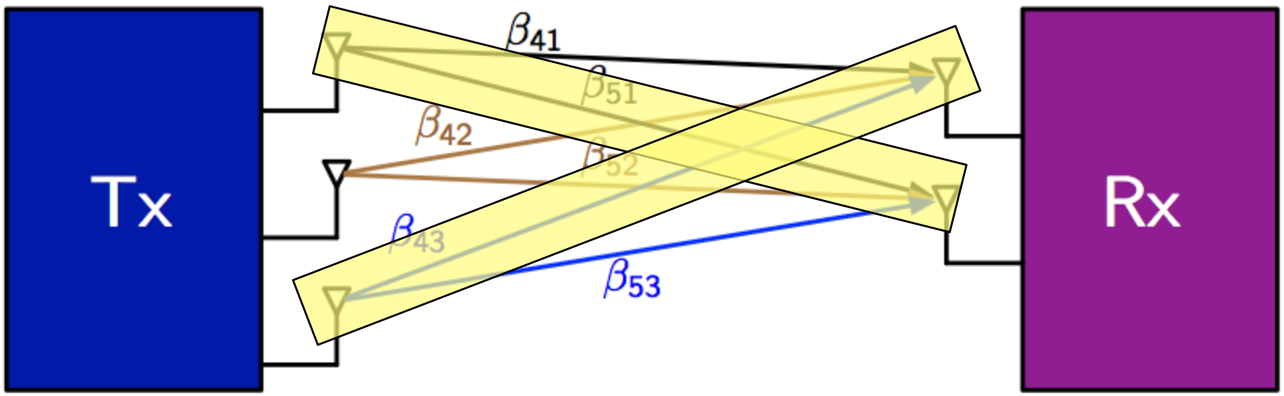

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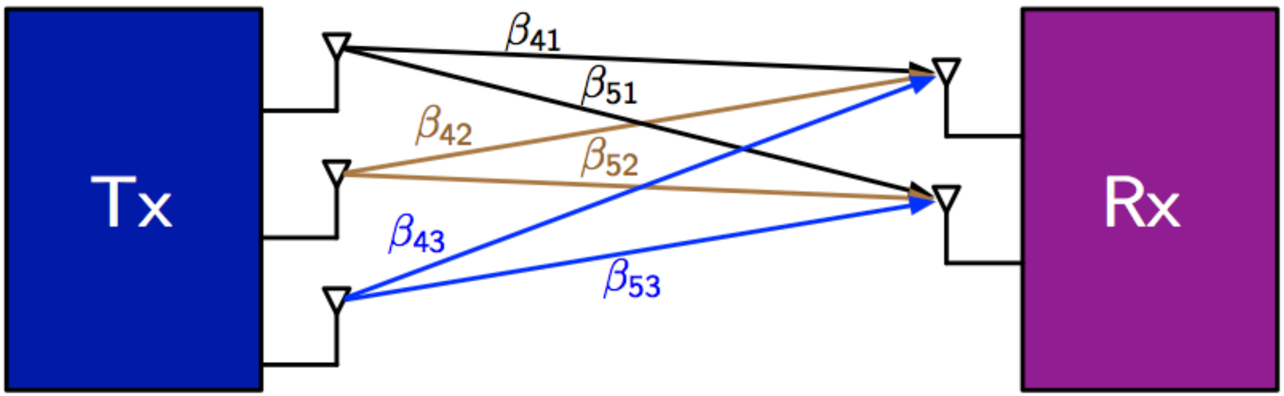

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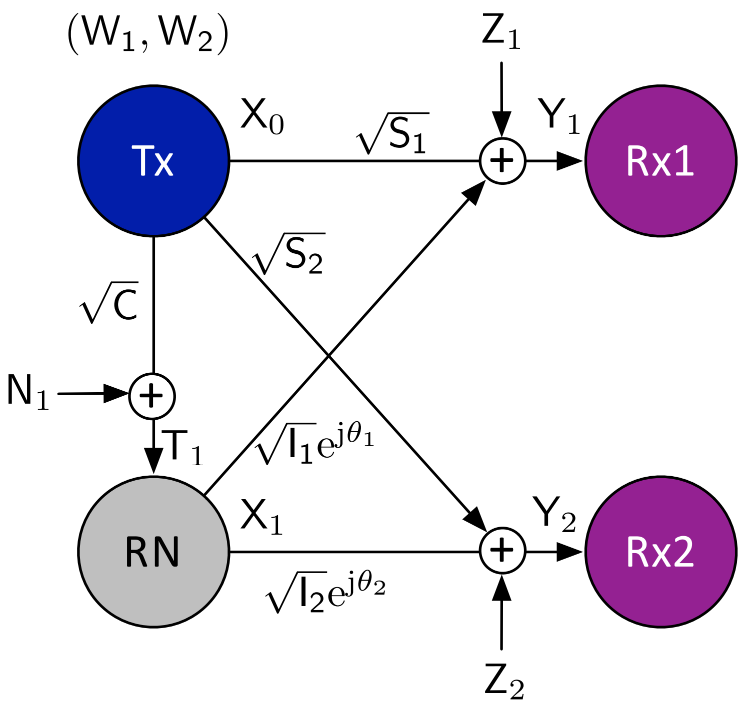
Example: FD SISO BC+relay



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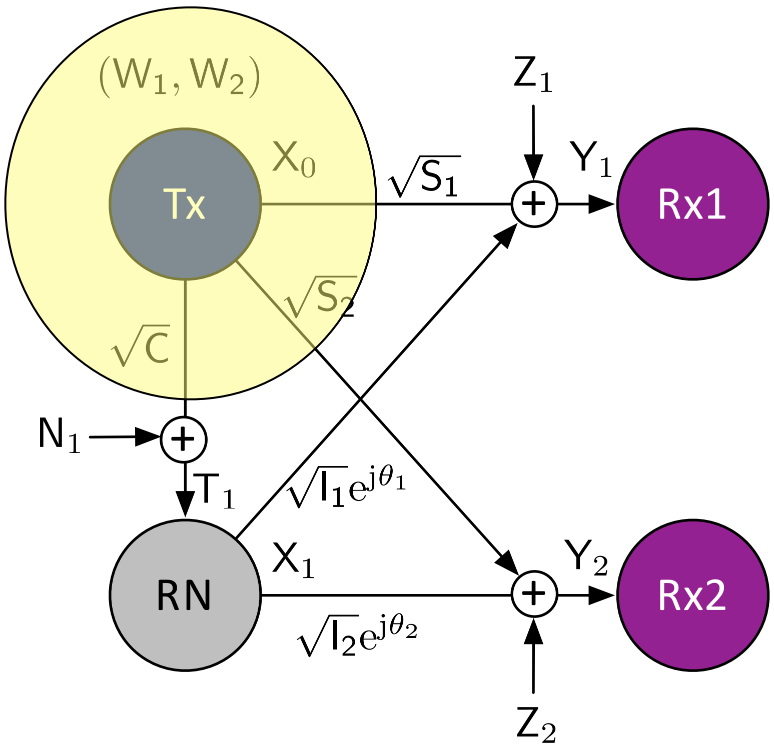

$$R_1 + R_2 \leq I\left(X_0; Y_R, Y_1, Y_2 | X_1\right)$$
$$\longleftrightarrow \max\left\{\beta_{10}, \beta_{20}, \delta\right\}$$

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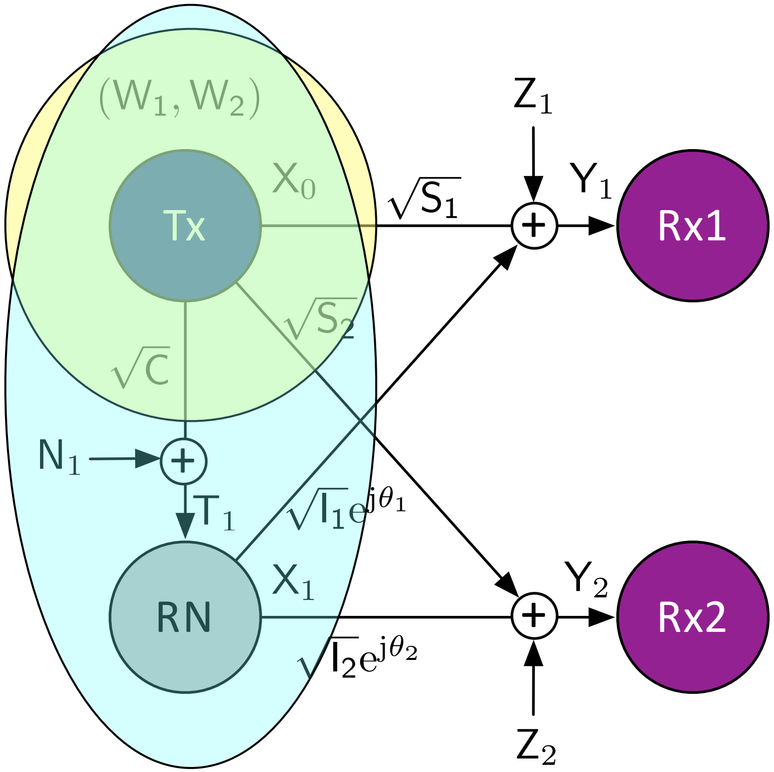
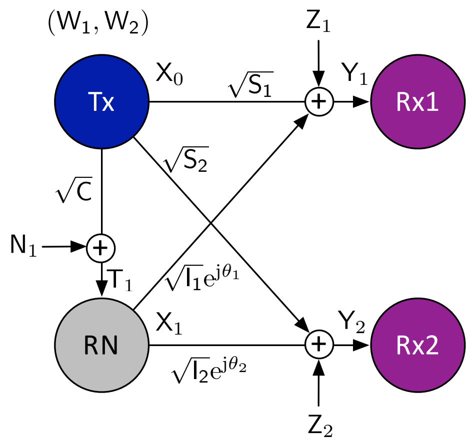
Example: FD SISO BC+relay



$$R_1 + R_2 \leq I\left(X_0; Y_R, Y_1, Y_2 | X_1\right)$$
$$\longleftrightarrow \max\left\{\beta_{10}, \beta_{20}, \delta\right\}$$

$$R_1 + R_2 \leq I\left(X_0, X_1; Y_1, Y_2\right)$$
$$\longleftrightarrow \max\left\{\beta_{10} + \beta_{21}, \beta_{11} + \beta_{20}\right\}$$

Example: FD SISO BC+relay



$$\min\left\{\max\{\beta_{10},\beta_{20},\delta\}, \max\{\beta_{10}+\beta_{21},\beta_{12}+\beta_{20}\}\right\}:$$

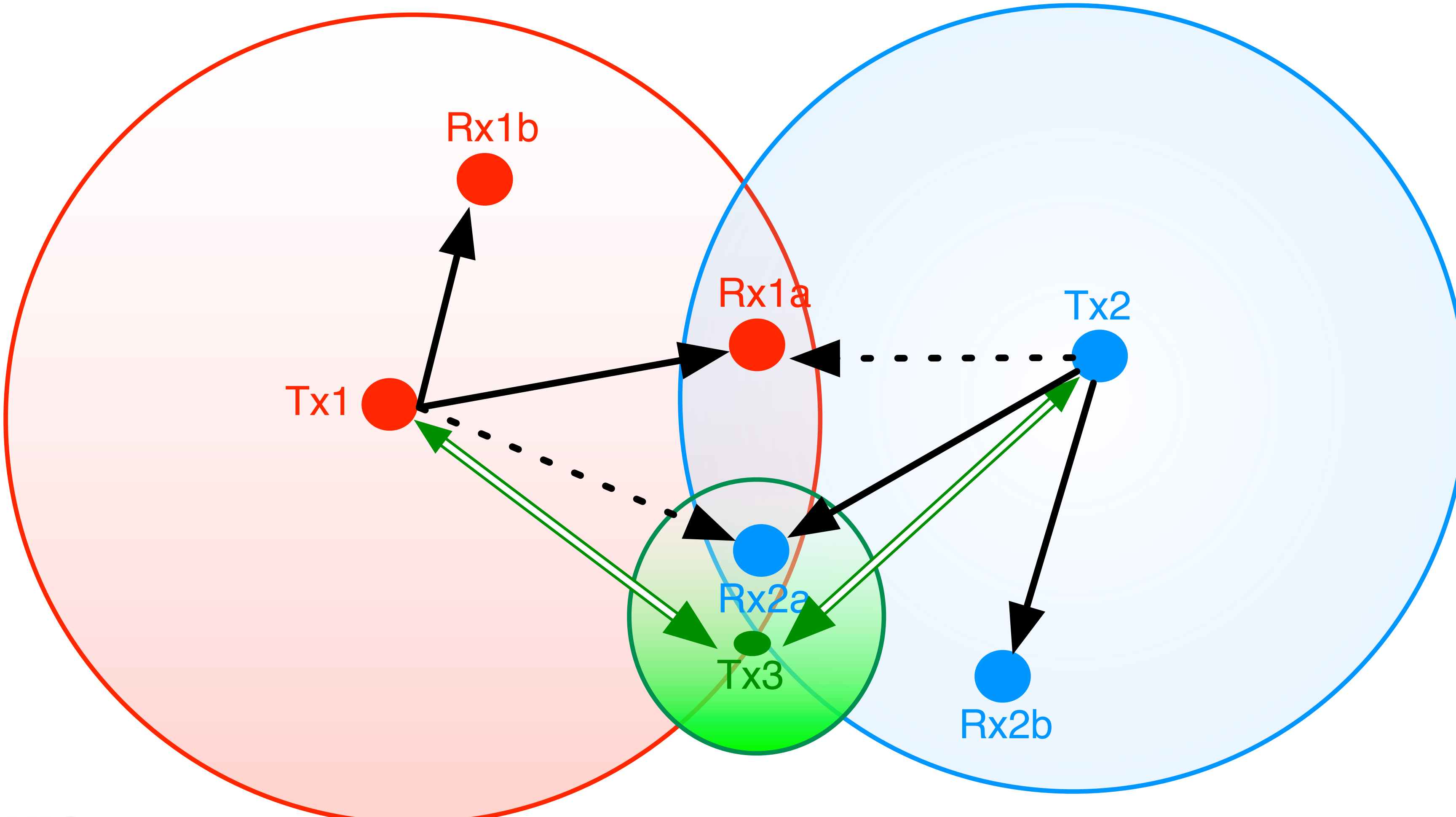
$$0 \le \delta \le \max\{\beta_{10},\beta_{20}\} : \text{serve 'best' user without relay}$$

$$\max\{\beta_{10},\beta_{20}\} < \delta \le \max\{\beta_{11},\beta_{21}\} : \text{serve 'best' user with relay}$$

$$\max\{\beta_{11},\beta_{21}\} < \delta \le \max\{\beta_{10}+\beta_{21},\beta_{12}+\beta_{20}\} : \text{serve both users}$$

$$\max\{\beta_{10}+\beta_{21},\beta_{12}+\beta_{20}\} < \delta : \text{serve both users} == \text{MISO BC}$$

HetNets: BCIC + relays



Recipe

- Take your favorite outer or lower bound, possibly further upper or lower bound so as to only have channel gains
- Pre-log == gDoF == MWBM
- The overall channel matrix <u>must</u> be full rank (i.e., approximation too “crude” to capture small variations in channel gains, example $\mathbf{H} = \begin{bmatrix} (1+\epsilon)\sqrt{S} & \sqrt{S} \\ \sqrt{S} & \sqrt{S} \end{bmatrix}$)

Disclaimer

- SISO compound MAC + relays: NNC achieves 0.63 x 2 x N bits of cut-set bound
- SISO private msgs BC + relays: can achieve O(N log(N)) bits of cut-set bound
- IC (+ relays): open, and cut-set bound known to be insufficient ....

Part II

Mixed Inputs and Treat Interference as Noise:

mixed input refers to a random variable that is a mixture of a continuous and a discrete part, i.e., a Gaussian and a uniform PAM

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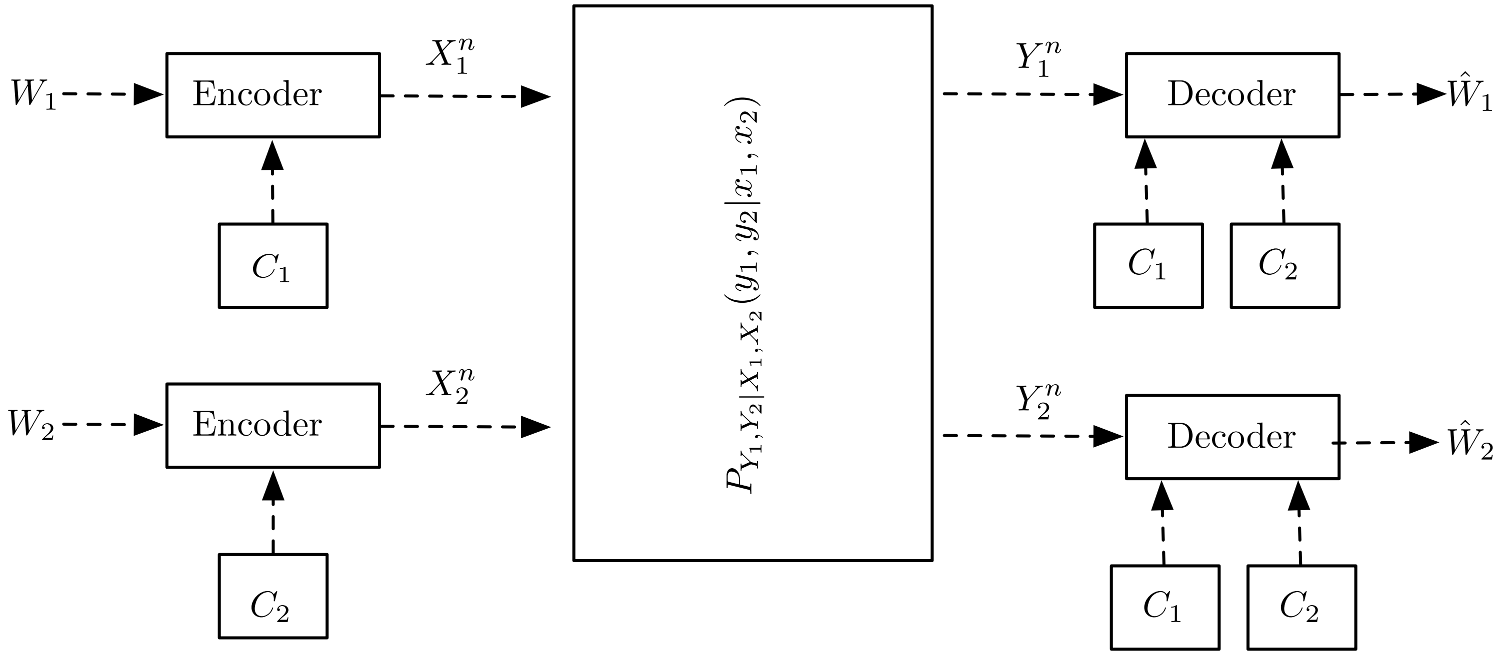
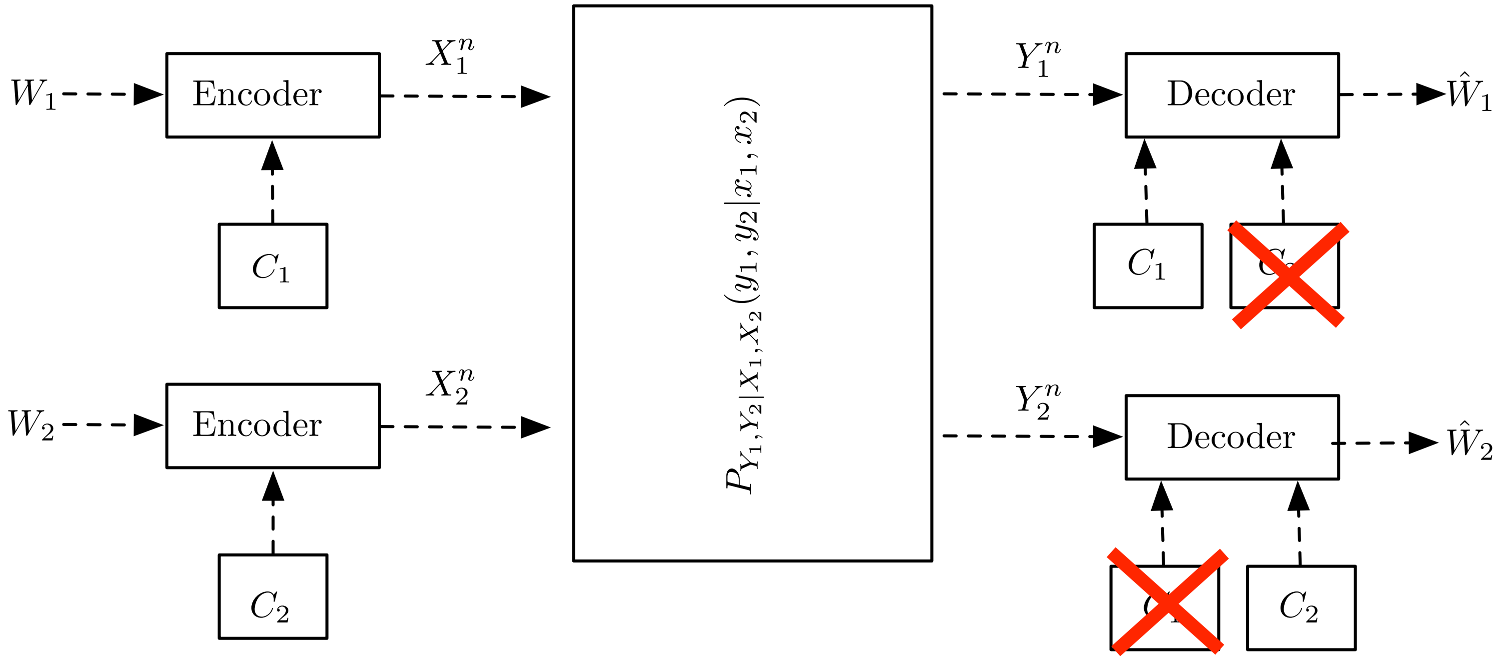
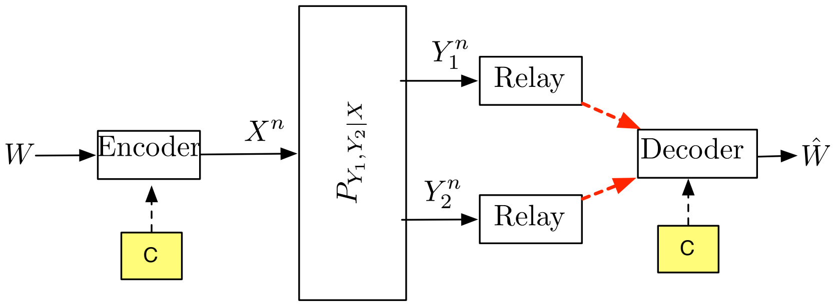
Oblivious Processing



Oblivious Processing



Past Work

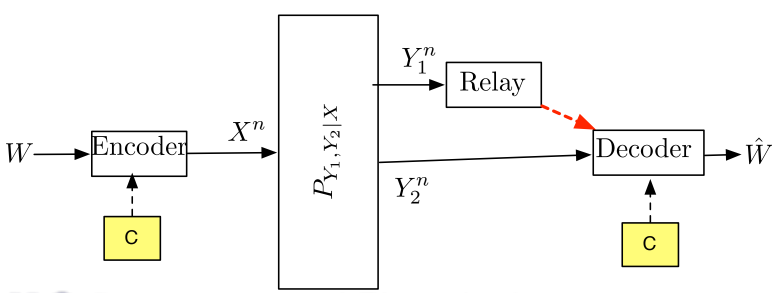


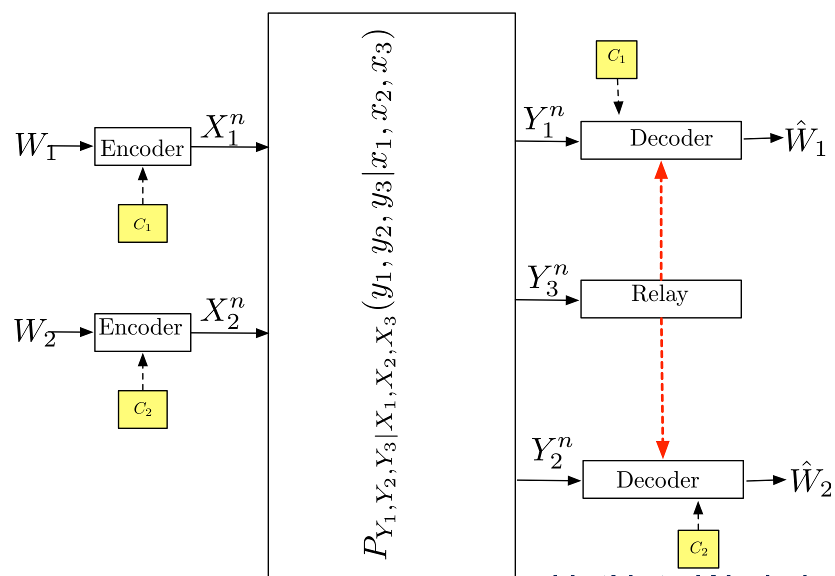
A. Sanderovich, S. Shamai, Y. Steinberg, and G. Kramer, "*Communication via decentralized processing*," IT July 2008.

1. Upper and lower bounds, which coincide for deterministic channels
2. Gaussian noise: optimizing input unknown
3. Gaussian noise: example where BPSK outperforms Gaussian inputs

O. Simeone, E. Erkip, and S. Shamai, "On codebook information for interference relay channels with out-of-band relaying," IT May 2011.

1. Primitive relay channel: capacity with compress forward
2. IC+R+Oblivious receivers: capacity with compress forward and TIN
3. Gaussian noise: optimizing input unknown





Past Work (discrete inputs)

- Y. Wu and S. Verdu, "The impact of constellation cardinality on Gaussian channel capacity," Allerton 2010 (point to point)
- E.Calvo *et al* "On the totally asynchronous interference channel with SU receivers," ISIT 2009
- E.Abbe and L.Zheng, "A coordinate system for Gaussian networks," IT Feb. 2012
- Achievable for any (i-stable) IC $R_k \leq I(X_k;Y_k),\ k \in [1:K]$
- Continuous inputs are "bad interferers" -- especially if one treats them as noise

Main Tool

$$Z_G \sim N(0,1) \text{ independent of } X_D \sim \text{discrete}:$$

$$\mathsf{I_d}\left(N, d^2_{\min(X_D)}\right) \le I(X_D; X_D + Z_G) \le \frac{1}{2}\log\left(\min\left(N^2, 1+\mathcal{E}_{X_D}\right)\right)$$

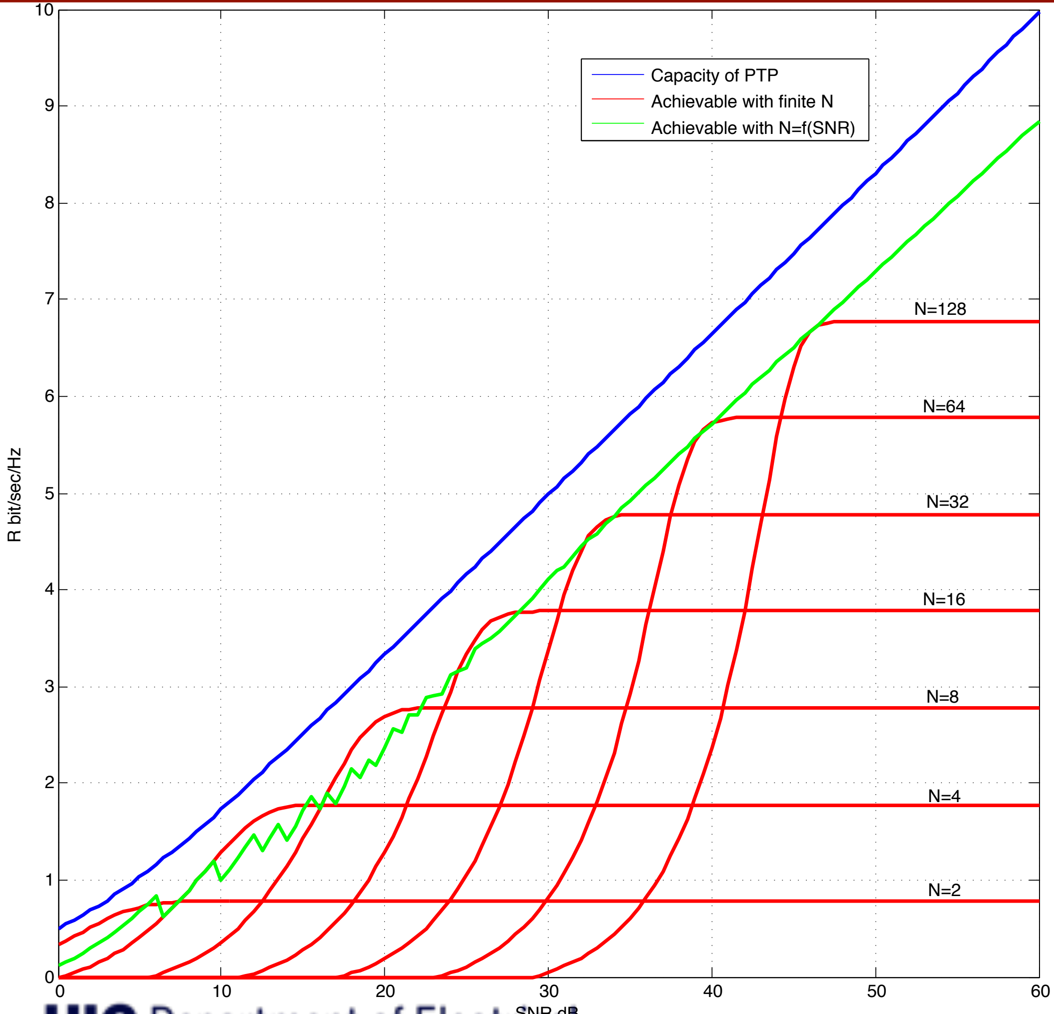
$$\mathsf{I_d}(n,x) := \left[\log(n) - \frac{1}{2}\log\left(\frac{e}{2}\right) - \log\left(1+(n-1)e^{-4x}\right)\right]^+$$

$$N^2 = 1+\mathcal{E}_{X_D}, \quad N\, e^{-4d^2_{\min(X_D)}} < \text{constant}$$

$$I(X_D; X_D + Z_G) = \frac{1}{2}\log\left(1+\mathcal{E}_{X_D}\right) - \text{constant}$$

Lower bound holds for any constellation but may be arbitrary lose for a specific one, i.e., for PAM.

Example: AWGN



$$Y = \sqrt{\mathsf{SNR}}\, X + Z :$$

$$Z \sim \mathcal{N}(0,1), \ \ X \sim \mathrm{PAM}\,(N)\,,$$

$$d^2_{\min(X)} = \frac{12}{N^2 - 1},$$

$$N = \left\lfloor \sqrt{1 + \mathsf{SNR}^{1-\epsilon}} \right\rfloor$$

$$\epsilon = \frac{\left[\frac{1}{2}\log\left(\frac{1}{6}\ln(\mathsf{SNR})\right)\right]^+}{\frac{1}{2}\log(\mathsf{SNR})}$$

$$\mathrm{gap} = \frac{\epsilon}{2}\log(\mathsf{SNR}) + \frac{1}{2}\log\left(8e\right)$$

Main Result

- Choice of inputs

$$X_i = \sqrt{1-\delta_i}\ X_{iD} + \sqrt{\delta_i}\ X_{iG}, \quad \delta_i \in [0,1],$$
$$X_{iD} \sim \mathrm{PAM}\left(N_i\right),\ X_{iG} \sim \mathcal{N}(0,1),$$
$$\text{where } X_{ij} \text{ are independent for } i \in [1:2], j \in \{D,G\}.$$

- Discrete part = \`common message'

$$N = \left\lfloor \sqrt{1+x^{1-\epsilon}} \right\rfloor$$

$$\epsilon = \frac{\left[\frac{1}{2}\log\left(\frac{1}{a}\ln(x)\right)\right]^+}{\frac{1}{2}\log(\mathsf{SNR})}$$

$$R = \frac{1}{2}\log(1+x) - \text{gap}$$

$$\text{gap} = \frac{\epsilon}{2}\log(\mathsf{SNR}) + \frac{1}{2}\log(8e)$$

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How about TDMA?

$$R = \sum_i \tau_i \frac{1}{2} \log\left(1 + \mathsf{SNR}_i\right)$$

$$\approx \log\left(\prod_i \sqrt{\mathsf{SNR}_i}^{\tau_i}\right)$$

$$\sqrt{\mathsf{SNR}_i} \longleftrightarrow N_i$$

$$N = \prod_i N_i^{\tau_i}$$

No need to time-share / coordinate: the same effect (up to a gap) can be obtained by varying the number of points of the discrete part

Recipe

- Common message <--> discrete input
- Private message <--> Gaussian input
- TIN is optimal to within log(log(SNR))
- No need of joint decoding
- No need of synchronous communication
- TDMA by appropriately varying number of points in the discrete part of the input

Thank you

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