Capacity and Scheduling in Heterogeneous Networks

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Joint Work with Sem Borst and Stephen Hanly

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Talk Summary

- Mobile Radio and the Spectrum Crunch
- Getting more Capacity and How much do we Have?
- Utility Schedulers
- Closing Remarks

Mobiles Past

An Entrepeneur Securing a Deal using an Early Mobile Phone

Mobiles Present

Progress toward Data, Apps - Location Based Information

Mobiles Future

Future User having Trouble with a Hotel Booking ....

Talk Summary

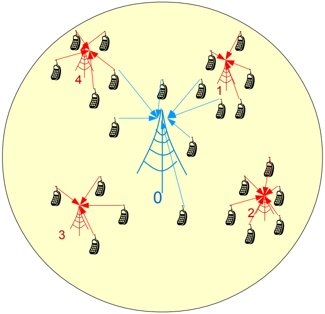
- A SnapShot Resource Allocation Problem
- A Continuous LP
- Capacity and Scheduling
- α-fair Utility Scheduling
- Stability Results
- Conclusions

Gaining Capacity using HetNets

- Small cells (pico/femto) to increase frequency reuse
  - Place in areas of poor coverage
  - Areas of traffic concentration - "Hot Spots"
- Adapt Network to Match Traffic Load

A Simplified HetNet Model

$L = 4$ picos - all users in range of macro and at most one pico
No Interference between Pico Cells



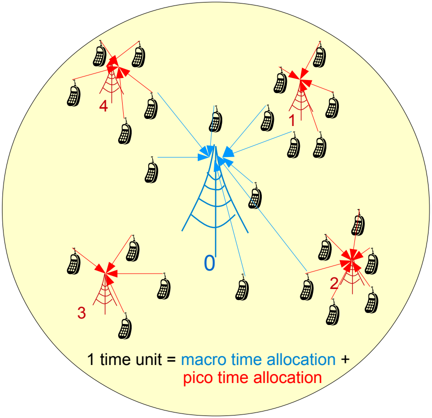
Flexible Allocation

- Time Share Spectrum
  - Macro Cell/Pico Cells
  - Use Almost Blanking SubFrames (fine granularity)
- Cell Range Expansion for Picos
  - Expand to cover more mobiles
  - Contract and send at Higher Rate

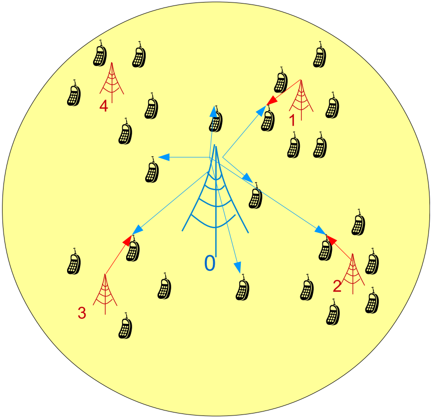
For following, see [1]

[1] S. Borst, S. Hanly, P. Whiting "Optimal resource allocation in HetNets", ICC, Budapest, Hungary, 2013.

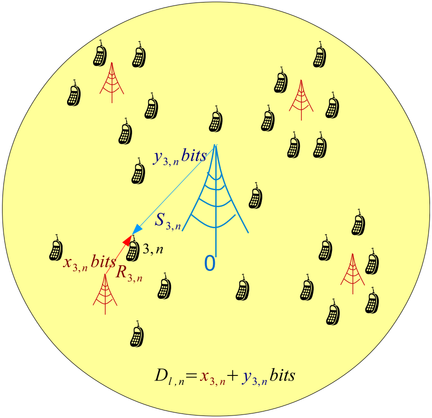
ABS Frames and Time Sharing



Mobiles can TimeShare Macro/Pico (Split)



Empty the Network !!



Solve the Following LP!

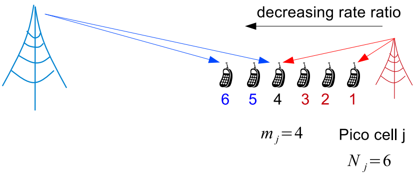
- The problem to be solved is the following linear program:

$$
\begin{aligned}
\min \quad & f + \sum_{l=0}^{L} \sum_{n=1}^{N_l} \frac{y_{l,n}}{S_{l,n}} \\
\text{sub} \quad & \sum_{n=1}^{N_l} \frac{x_{l,n}}{R_{l,n}} \le f \quad \forall l \\
& x_{l,n} + y_{l,n} \ge D_{l,n} \quad \forall l,\ \forall n = 1, 2, \ldots N_l \\
& f \ge 0, x_{l,n} \ge 0, y_{l,n} \ge 0 \quad \forall l,\ \forall n = 1, 2, \ldots N_l
\end{aligned}
$$

where f is the time allocated to the picocells.

Solution Structure

- $\rho_{l,n} := \frac{R_{l,n}}{S_{l,n}}$
- Order User - Decreasing in ρ
- Large $\rho \to$ pico, Small $\rho \to$ macro, $= \rho \to$ Split



Let's make the Problem Continuous ...

Continuous LP parameters

$\lambda_S \eta(d\xi) = \lambda(d\xi)$, $\eta(d\xi)$ probability density

$R_\ell(\xi), S_\ell(\xi)$ Phy. Rates Pico/Macro - Pico ℓ

$x_\ell(\xi), y_\ell(\xi)$ bit assignments at location ξ

D download file size (could be random, here fixed)

Largest λ_S for which network is stable?

Continuous LP

$$\min \qquad \tau = f + \sum_{\ell=1}^{L} \int \frac{y_\ell(\xi)}{S_\ell(\xi)} \lambda(d\xi) \tag{1}$$

$$\text{sub} \qquad \int \frac{x_\ell(\xi)}{R_\ell(\xi)} \lambda(d\xi) \le f \quad \forall \ell \tag{2}$$

where,

$$y_\ell(\xi) = D - x_\ell(\xi)$$

is the file constraint

Optimal solution

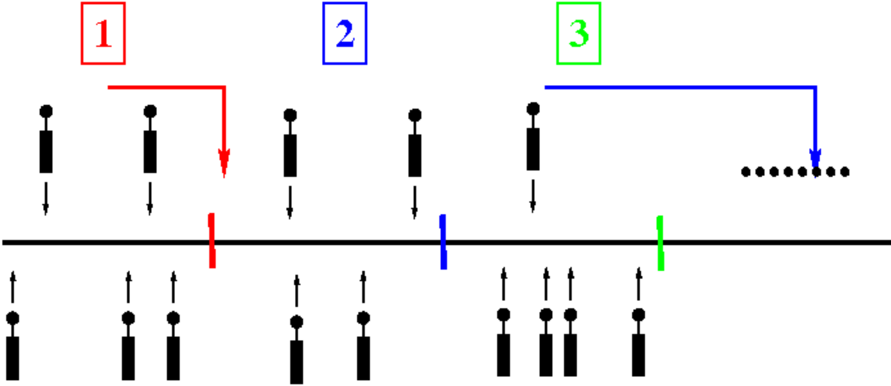
For some $\rho_1, \cdots, \rho_L > 0$,

$$x_\ell^*(\xi) = \begin{cases} D & \frac{R_\ell(\xi)}{S_\ell(\xi)} \geq \rho_\ell \\ 0 & \frac{R_\ell(\xi)}{S_\ell(\xi)} < \rho_\ell \end{cases} \tag{3}$$

$$f^* = \max_\ell \int \frac{x_\ell^*(\xi)}{R_\ell(\xi)} d\xi$$

If $\tau^* < 1$, $\exists$ a stable schedule $\cdots$

Bundling $[nT, (n+1)T),\ n \in \mathbb{N}_0$



Bundling Algorithm

- 1 $B := 1$, Wait until $n_B = 1$
- 2 Serve bundle B, starting $n_B T$
- 3 Let $f_B T$ completion slot for bundle B
- 3 $B := B + 1$, $n_B := \max\{f_B, B\}$
- 4 Go to 2

Bundling defines a $D/G/1$ queue, bundle delay $=: W_n$

$\tau < 1$, assumptions $\rightarrow \mathbb{E}[W_n]$ Uniformly Bounded
W_n satisfies Spitzer's identity,

$$\begin{aligned} \mathbb{E}[W_n] &= \mathbb{E}\left[\max_{k \leq n} S_k^+\right] \\ &= \sum_{k=1}^{n} \frac{1}{k} \mathbb{E}\left[S_k^+\right] \end{aligned}$$

$S_k \doteq X_k - kT$, X_k duration first k bundles

SLLN and Stability

Any bounded, measurable $v : S \to \Re_+$,

$$\frac{1}{T}\sum_{n=1}^{N_T} v(\xi_n(\omega)) \to \int_S v(\xi)\lambda(d\xi) \tag{4}$$

a.s. and in $\mathcal{L}_1$.

$$\upsilon_\ell^T(\omega) \doteq \frac{1}{T}\sum_{n=1}^{N_T} \frac{x_\ell(\xi_n(\omega))}{R_\ell(\xi_n(\omega))}$$

is UI, $\ell = 1, \cdots, L$.

$$\to f_T(\omega) = \max_\ell \upsilon_\ell^T(\omega)$$

is UI so that $\mathbb{E}[f_T] \to f^*$

$$\mathbb{E}[f_T] + \sum_{\ell=0}^{L} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T} \frac{y_\ell(\xi_n)}{S_\ell(\xi_n)}\right] \to \tau^* < 1$$

A schedule π is clearing if departure time $D_n^\pi(\omega) < \infty, \;a.s., \;\forall n$

Prop (Hanly, W.)

Let τ^* be optimal solution to the LP. If $\tau^* < 1$, $\exists$ a clearing schedule π with ergodic properties.
Also define $S_n^\pi(\omega) :=$ sojourn time nth mobile, then π satisifies,

$$\mathbb{E}\left[S_n^\pi(\omega)\right] < \overline{S} < \infty \tag{5}$$

Converse Holds as Well!

Continuous LP $\tau^* > 1 \;\rightarrow\;$ No Stable Schedule

Let π be any clearing schedule. Define $V_T^\pi(\omega)$ to be network time needed to clear mobiles arriving in $[0, T]$

Prop (Hanly, W.)

Let τ^* be the solution to the continuous LP. Suppose that $\tau^* > 1$ then there is a fixed constant $\eta > 0$, such that for all π

$$\liminf_T \frac{V_T^\pi(\omega)}{T} = 1 + \eta$$

almost surely.

Proof Sketch I

Arrivals in $[0, T]$ supposed to arrive at time 0. Apply discrete LP with outcome $V_T^{(LP)}(\omega)$

Prop

$\forall\omega$ and for all clearing schedule π,

$$\liminf_T \frac{V_T^{(LP)}(\omega)}{T} \leq \liminf_T \frac{V_T^{\pi}(\omega)}{T} \tag{6}$$

Proof Sketch II: Discretise Arrivals using Rate Ratios ρ_ℓ

Given $\varepsilon > 0$, choose intervals,

$N_T^{(\ell,n)}$ arrivals in interval n for pico ℓ mean $m_\ell(n)$
For all $0 < \delta < 1/2$ there exists $I_{n,\ell} > 0$

$$\mathbb{P}\left\{ \frac{1}{T} N_T^{(\ell,n)} \notin [(1-\delta)m_\ell(n), (1+\delta)m_\ell(n)] \right\} \leq e^{-TI_{n,\ell}} \qquad (7)$$

Borel-Cantelli implies $\exists T_E$ all arrivals close to expectation, $\forall T > T_E$

Proof Sketch III

Finite set $\mathcal{A}$ of rate ratio policies,

$$
\begin{aligned}
\liminf_T \frac{V_T^{(LP)}}{T} &\geq \liminf_T \inf_{a\in\mathcal{A}} \frac{V_T^a}{T} - \frac{L\varepsilon D}{\underline{R}} && (8)\\
&= \inf_{a\in\mathcal{A}} \liminf_T \frac{V_T^a}{T} - \frac{L\varepsilon D}{\underline{R}} && (9)\\
&\geq (1+\eta) - \frac{L\varepsilon D}{\underline{R}} && (10)
\end{aligned}
$$

Utility Scheduling and Stability

Modelling Assumptions

- Discrete set - location k in cell ℓ - (k, ℓ), $k = 1, \cdots, K_l$

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- Independent Poisson streams, $\lambda_k^{(\ell)} > 0$
- Physical Rates $R_k^{(\ell)}$ pico, $S_k^{(\ell)}$ macro

Time Sharing

Time Sharing Vector $(\mathbf{a}, \mathbf{b})$

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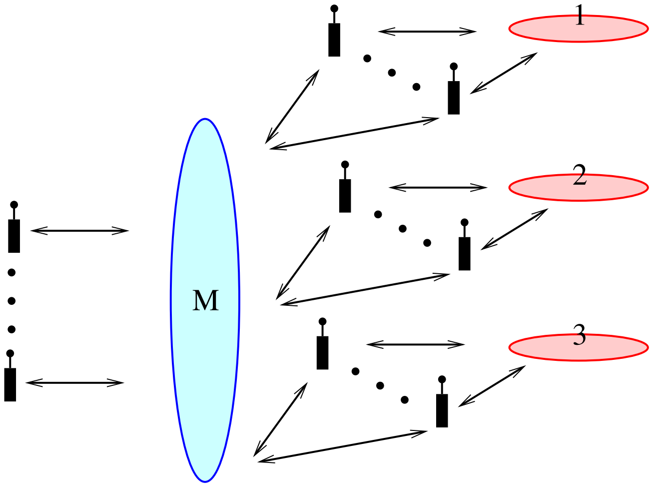
Feasibility constraints,

$$\sum_{k=1}^{K^{(\ell)}} a_k^{(\ell)} + \sum_{m=0}^{L} \sum_{k=1}^{K^{(m)}} b_k^{(m)} \leq 1, \ \forall \ell. \tag{11}$$

with throughput,

$$T_k^{(\ell)} = a_k^{(\ell)} R_k^{(\ell)} + b_k^{(\ell)} S_k^{(\ell)} \tag{12}$$

Processor Sharing Model for a HetNet



Stability Region Λ_0

$$\mathcal{T} \doteq \{(\mathbf{a}, \mathbf{b}) : (\mathbf{a}, \mathbf{b})\,,\ \textit{feasible}\}\,,$$

$$\Lambda \doteq \cup \{\mathbf{T}(\mathbf{a}, \mathbf{b}) : (\mathbf{a}, \mathbf{b}) \in \mathcal{T}\}$$

Then,

$$\Lambda_0 \doteq \{\boldsymbol{\lambda} : \exists \boldsymbol{\varepsilon} > 0, \boldsymbol{\lambda} + \boldsymbol{\varepsilon} \in \Lambda\}$$

Stable scheduler exisits iff $\boldsymbol{\lambda} \in \Lambda_0$

Continuous Time Markov Processes

$$\mathbf{N}(t) \doteq \left(\mathbf{N}^{(0)}(t), \cdots, \mathbf{N}^{(L)}(t)\right) \in \prod_{\ell} \mathbb{N}_0^{K^{(\ell)}} =: \mathcal{N}$$

Arrivals, rate $\lambda_k^{(\ell)}$,

$$\left(\mathbf{N}^{(0)}, \cdots, \mathbf{N}^{(L)}\right) \to \left(\mathbf{N}^{(0)}, \cdots, \mathbf{N}^{(L)}\right) + \left(0, \cdots, \mathbf{e}_k^{(\ell)}, \cdots, 0\right)$$

Departures policy θ, in state $\mathbf{n} \in \mathcal{N}$, rate $\mathbf{T}\left(\mathbf{a}^{\theta}(\mathbf{n}), \mathbf{b}^{\theta}(\mathbf{n})\right)$

$$\left(\mathbf{N}^{(0)}, \cdots, \mathbf{N}^{(L)}\right) \to \left(\mathbf{N}^{(0)}, \cdots, \mathbf{N}^{(L)}\right) - \left(0, \cdots, \mathbf{e}_k^{(\ell)}, \cdots, 0\right)$$

A Static Utility Optimization Problem

$$U(\mathbf{a}, \mathbf{b}) \doteq \sum_{\ell=0}^{L} \sum_{k} N_k^{(\ell)} U_\alpha \left(\frac{T_k^{(\ell)}(\mathbf{a}, \mathbf{b})}{N_k^{(\ell)}} \right) \tag{13}$$

α-fair utilities

$$U_\alpha(\cdot) = (1-\alpha)^{-1} x^{1-\alpha}, \ \alpha \in (0, \infty)$$

For solution to above,, see [2]

[2] S. Borst, S. Hanly, P. Whiting "Throughput Utility Optimization in HetNets", VTC, Dresden, Germany, 2013.

Prop (Hanly, W.)

Suppose $\boldsymbol{\lambda} \in \Lambda_0$. *Then* $\forall \alpha > 0$ *the Markov Process defined by* α*-fair scheduling is positive recurrent so that*

$$\mathbb{P}\left\{\mathbf{N}(t) = \mathbf{N}\right\} \to \pi^{\alpha}(\mathbf{N}) \text{ as } t \to \infty \tag{14}$$

Moreover limiting α *moments exist; that is, for all* (k, ℓ),

$$\mathbb{E}_{\pi^{\alpha}}\left[\left(N_k^{(\ell)}\right)^{\alpha}\right] < \infty \tag{15}$$

Proof Sketch

As demonstrated in [3]

$$L(\mathbf{N}) \doteq \sum_{\ell=0}^{L} \sum_{k=1}^{K^{(\ell)}} \left\{\lambda_k^{(\ell)}\right\}^{-\alpha} \frac{\left\{N_k^{(\ell)}\right\}^{1+\alpha}}{(1+\alpha)} \tag{16}$$

is a Lypuanov function

Let $\mathbf{N}(n)$ jump chain sequence of the uniformized Markov process, then,

$$L(\mathbf{N}(n))$$

has supermart. property outside a compact set.

[3] T. Bonald and L. Massouliè "Impact of Fairness on Internet Performance", ACM SIGMETRICS Performance Evaluation Review, Vol. 29, No. 1, pp 82–91, 2001.

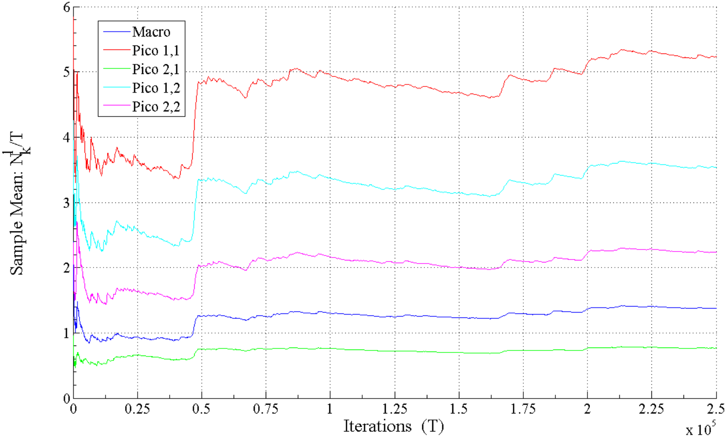
Example: Proportional Fair Scheduler

$$U_{\mathbf{N}} \doteq \sum_{\ell=0}^{L} \sum_{k=1}^{K^{(\ell)}} N_k^{(\ell)} \log \frac{T_k^{(\ell)}}{N_k^{(\ell)}} \tag{17}$$

Quadratic Lypuanov function L,

$$L(\mathbf{N}) \doteq \frac{1}{2} \sum_{\ell=0}^{L} \sum_{k=1}^{K^{(\ell)}} \frac{\left\{ N_k^{(\ell)} \right\}^2}{\lambda_k^{(\ell)}} \tag{18}$$

Numerical Results



Conclusions

- Traffic Capacity Determined by LP
- Fixed Schedule Stable
  - Estimate η
  - Estimate $R_\ell(\xi), S_\ell(\xi)$
  - Infer Capacity
- Results extend to more general networks

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- Traffic Capacity Determined by LP
- Fixed Schedule Stable
  - Estimate η
  - Estimate $R_\ell(\xi), S_\ell(\xi)$
  - Infer Capacity
- Results extend to more general networks
- α-fair Utility Scheduler maximally stable
- Equilibrium Moments shown to exist depending on α
- Results extend to periodic schedulers

Thanks!