

ARC Final Report - Discovery-Project - (DP)

In the interests of promoting awareness of the benefits of publicly-funded research, the ARC may draw on the material you provide about your research for reporting and promotional purposes.

Part A. PROJECT IDENTIFICATION

A1 Program DP

A2 Project ID DP0558372

A3 Administering organisation Macquarie University

A4 First year funded 2005

A5 Title of funded project

Foundations of higher dimensional homological algebra

A6 1st named Participant (note: if these details are incorrect amend your GAMS record)

<i>Title</i>	<i>Dr</i>
<i>First Name</i>	<i>Michael</i>
<i>Family Name</i>	<i>Batanin</i>
<i>Gams ID</i>	<i>J99415</i>
<i>Organisation</i>	<i>Macquarie University</i>
<i>Participant role</i>	<i>CI</i>
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A7 Other Participants

Title	FirstName	Family Name	GAMS ID	Organisation	Participant rol
no other participants named in the application.					

A8 For this Project, total ARC Funding requested 261046

A9 For this Project, total ARC Funding received 156143

A10 Number of years for which Funding was received 3

Part B PROJECT DESCRIPTION AND OBJECTIVES

B1 100 word Project summary (from the original project application)

'Homotopical Mathematics' is a term introduced recently to designate a rapidly developing methodology. It is based on the substitution of set theoretical notions by homotopy theoretical notions in a large part of mathematics relevant to geometry and physics. This approach has already produced spectacular applications in algebraic geometry, topology and mathematical physics. Homological algebra lies at the heart of this approach, yet its further development and application require clear and consistent foundations. In our project we intend to construct such foundations, using methods of Higher Category Theory. As an outcome, proof of important conjectures from both areas will arise naturally.

B2 Summary of original objectives of project

The project aimed to answer or developed approaches to some important questions in foundations of Higher Category Theory and Homotopy Theory:

- what do A_{∞} -categories form?
- what is higher braided A_{∞} -category?
- what is the "derivation process" of a higher categorical structure?

Possible applications were:

- a proof of Breen-Baez-Dolan stabilisation hypothesis,
- a conceptual proof of Deligne's conjecture and its generalisation,
- new algebraic models of homotopy type and n-fold loop spaces,
- explicit description of combinatorics of n-fold loop spaces,
- applications to comparisons of different approaches to Higher Category Theory,
- applications to strong shape theory.

Part C PROJECT OVER DURATION OF FUNDING

- C1 Were there significant changes to the Project or the objectives? Yes
If yes, please briefly describe the changes and explain why they were made.

For example, changes may have been made because the ARC funding was less than the requested funding.

The actual ARC funding was 60% of requested amount. This affected the length of the employment of my research assistant (Mark Weber) and also led to changes of the budget (approved by ARC) after Weber's earlier than planned resignation. As a consequence some of the objectives of the project were scaled back. More specifically, applications to strong shape theory were not considered and some other applications (derivation processes and comparison of different approaches to Higher Category Theory) were not developed in full generality.

- C2 Did anything affect the satisfactory and timely progress or completion of the Project? Yes
If yes, please briefly describe what happened.

My research assistant Mark Weber resigned after only 12 months of employment and I had to reconsider my budget to be able to complete the project. I got approval from ARC to spend the rest of Weber's salary for my trip to Europe in 2007-2008 and inviting Weber, Cisinski and Markl to Macquarie University in 2008. Since these visits could not be organised in three months left in 2007 the timing of the project has been extended until the end of 2008.

This variation of budget allowed me to complete an important paper with Mark Weber on multitensors generated by higher operads (this paper is now accepted for publication) and also to continue this work with Mark Weber and Cisinski (a joint paper is in the final stage of writing). My visit to Paris in 2008 and Cisinski and Wever visit to Sydney in August-September 2008 also initiated a development of a large research program concerning quasicategories, weak n-categories and comparisons of different approaches to higher categories. This research program will involve me, Weber (Paris 7), Cisinski (Paris 13), Berger (Nice), Joyal (Montreal). Due to the variation of the budget I also was able to visit Martin Markl in Prague (and invite him to Sydney in December 2008) and Clemens Berger in Nice. As a result we made a serious progress (in collaboration with Berger and Markl) in understanding of the natural action of higher operads on Hochschild complexes. This work generalises and considerably clarifies the earlier works on Deligne's conjecture by McClure-Skith, Berger-Fress, Kontsevich-Soibelman, Tamarkin and Markl. A joint paper with Markl and a joint paper with Berger on this subject have been submitted and another paper (joint with Berger and Markl) is close to completion.

- C3 Fellows on Team Projects - Briefly describe the Fellow's contribution to the Project
(ARC Fellows, who are Participants in a team of investigators but not the first-named Participant (Chief Investigator of Fellow), should contribute to this question)

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Part D PROJECT OUTCOMES AND IMPACTS

D1 Did the project meet its initial objectives or approved revised objectives? Yes

D2 Describe briefly the significance, results and outcomes of the project.

Most of the objectives of the project were achieved.

The most significant achievements are:

1. The development by CI of the theory of internal algebras of cartesian monads (a far reaching development of a microcosm principle of Baez and Dolan) and its application to understanding the Fulton-MacPherson compactification of configuration spaces and mysterious incompatibility of operadic and cellular structures of Fulton-MacPherson operad (famous Tamarkin counterexample). (A paper is published in "Advances in Mathematics".)

2. The solution by CI of the 40 years long standing problem of combinatorial characterization of n -fold loop spaces analogous to Stasheff's classical characterization of 1-fold loop spaces. (A paper is published in "Advances in Mathematics".)

3. The introduction by CI of higher braided operads and higher braided objects (a paper is published in "Journal of Noncommutative Geometry"). It answers a question about the existence of such a category and allows to formulate a corresponding recognition principle for n -fold loop spaces for arbitrary n . These were 40 years long standing questions. The answer was known only for $n=1,2$ or infinity. These results also show a clear way for proving a generalisation of Breen-Baez-Dolan stabilisation hypothesis one of the major hypotheses in higher category theory.

4. The full combinatorial description of opetopes in the joint work of CI, Kock, Joyal and Mascari (a paper is published in "Advances in Mathematics").

5. A description of the algebras of higher operads in terms of generalised enrichment is given in a joint paper by CI and Mark Weber (accepted for publication in "Appl. Categorical structures".)

6. A theory of tensor products of higher dimensional algebras is developed. As a byproduct a comparison theorem between Trimble's and Batanin's higher categories is obtained. A paper (joint with Weber and Cisinski) is submitted to "Advances in Mathematics".

7 A notion of natural operation on Hochschild complex is introduced. The full description of all natural operations is obtained. A new family of small categories, crossed interval groups, is discovered and an important generalisation of Deligne's conjecture is proved in a joint work of CI and Markl (a paper is submitted to "Geometry and Topology".)

7. A new combinatorial operad, the lattice path operad, and its cyclic version is found in a joint work with Berger and Markl. This operad is simple and generalizes and clarifies the earlier works of McClure-Smith, Berger-Fress, Tamarkin, Kontsevich-Soybelman and Joyal-Street. A joint paper with Berger is published in "Contemporary Mathematics" and a sequel paper (joint with Berger and Markl) is accepted for publication.

8. Mark Weber during his employment as my research assistant finished and published two long papers on 2-toposes and familial 2-functors. These papers quickly became quite influential forming part of the foundations of some important ongoing work by Cisinski (Paris 13), Maltsev (Paris 7) and Berger (Nice Sophia-Antipolis) which has a potential to completely clarify the relation between Batanin's weak omega-groupoids and homotopy types.

D3 Did the project lead to exciting new research directions, innovations and/or collaborations, and/or lay the foundations for new research and/or new partnerships? Yes
If yes, please describe briefly how.

This project led to the following new directions:

1. The theory of internal algebras developed by CI opens a perspective

for studying and describing explicitly the combinatorics of many important adjunctions.

The use by CI of compactification of configuration spaces open the way to understand the coherence for other important algebraic theories. One important application is the proof of Deligne's conjecture for Hochschild chains and cochains. This is currently a joint project with Tamarkin (Northwestern University, USA) and Dolgushev (Riverside University in California, USA).

2. The introduction of higher braided operads and higher braided objects by CI led to a new generalised form of Breen-Baez-Dolan stabilisation hypothesis. This result uses the full force of machinery of Cisinski (Paris XIII, France) and will be studied in close collaboration with him.

3. The earlier results by CI on higher fundamental groupoids and the work of CI, Weber and Cisinski on multitensors led to a close collaboration with Cisinski (Paris XIII), Berger (l'Universite de Nice, France) and Weber (Paris VII, France). As a result of this collaboration a large research program on quasi n-categories, homotopy types and weak n-categories was born. It should lead to the full understanding of the relationship between different approaches to Higher Category Theory and modelling of homotopy types. This research program will involve collaboration between CI, Berger, Cisinski, Weber, Maltsiniotis (Paris VII, France), Joyal (L'Universite de Quebec in Montreal, Canada).

3. The work on lattice path operad by CI and Berger and generalised Deligne's conjecture by CI and Markl (Academy of Science, Czech Republic) led to a close collaboration between us. There are many new directions which this work opened up.

4. The techniques of polynomial functors developed in the joint paper with Kock, Joyal and Mascari is innovative. We will continue our collaboration in this direction.

5. Influenced by our conversation, Tamarkin answered an important question "what do DG-category form ? " and gave the first conceptual and clear proof of Deligne's conjecture using my techniques of 2-operads and my theorem on symmetrisation of 2-operads. His construction was, in its turn, the primary source of inspiration for CI and Berger in our discovery of the lattice path operad. This opened a way for generalization of Tamarkin's construction for higher dimensions. This is a subject of our ongoing collaboration with Berger, Markl and Tamarkin.

D4 Are there identifiable national benefits—including economic, social, cultural and/or environmental contributions—resulting from this project? No
If yes, please describe briefly the national benefits.

Part E RESEARCH COLLABORATION

E1

Provide details of any collaborations or partnerships the research involved or led to – for each, as appropriate, list the names of the partner organisation, its type and country of location.

Name of Inst/Org	Type of Inst/Org	Country (if not Australia)
Paris 13	University	France
Paris 7	University	France
L'Universite de Nice	University	France
Max Planck -Institut fur Matematik	Research institute	Germany
Centre de Recerca Matematica	Research institute	Spain
l'Universite de Quebec a Montre	University	Canada
Northwestern University	University	USA
Mathematical Institute of the Academy	Research institute	Czech Rep.

Part F REPORT ON COLLABORATION BY INDUSTRY PARTNER/s

To be completed and certified by each Industry Partner.

You may attach a more extended statement if necessary.

Part F is not applicable to DP scheme.

Part G PROJECT OUTPUTS

G1 Field of research

<i>RFCD code</i>	<i>% weighting</i>
230104	80
230112	20

G2 Socio-economic objective

<i>SEO code</i>	<i>% weighting</i>
780101	100

G3 Publications and other academic outputs

Enter the number of outputs in each category. For programs other than LIEF, enter full details; for publications, include 'published' and 'in press' publications, but exclude 'forthcoming' and 'submitted' work.

Number

A1 Book—authored research

A2 Book—authored other

A3 Book—edited

A4 Book—revision/new edition

A4 Book—translation

B Book chapter

C1 Journal article—articles in scholarly refereed journal

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1. Batanin M., The symmetrisation of n -operads and compactification of real configuration spaces, Adv. Math. (211), (2007), pp. 684-725.

2. Batanin M., The Eckmann-Hilton argument and higher operads, Adv. Math. (217)(2008), pp. 334-385.

3. Batanin M., Locally constant n -operads as higher braided operads, Journal of Noncommutative Geometry, v.4(2010), pp.237-263

4. Batanin M, Weber M., Algebras of higher operads as enriched categories, Applied Categorical Structures, 38pp, accepted in November 2008.

5. Weber M., Yoneda structures from 2-toposes, Applied Categorical Structures, Vol. 15, p259-323, 2007.

6. Weber M., Familial 2-functors and parametric right adjoints, Theory and Applications of Categories 18, p665-732, 2007,

7. Batanin M., Berger C., The Lattice path operads and Hochschild cochains, Cont. Math. AMS, v.504, 2009, pp.21-52.

8. Batanin M., Berger C., Markl M. Operads of natural operations I: Lattice paths, braces and Hochschild cochains Accepted for publication in Proceedings of Soc. Math. France in series Seminaires et Congres, 2010

9. Batanin M., Joyal A., Kock J., Mascari J.-F., Polynomial functors and opetopes Advances in Mathematics 224 (2010), pp. 2690-2737

C2 Journal article—other contribution to refereed journal

C3 Journal article—non-refereed article

C4 Journal articles—letter or note

D Major reviews

E1 Conference—full written paper-refereed proceedings

E2 Conference—full written paper-non-refereed proceedings

E3 Conference—extract of paper

E4 Conference—edited volume of conference proceedings

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Batanin M. A.,Davydov, M.Johnson, S.Lack, A.Neeman, Categories in Algebra, Geometry and Mathematical Physics, Contemp. Mathematics, 480 p. (431)(2007)

E4 Conference—unpublished presentation

F Audio-visual recording

G Computer software

H Designs

J1 Major creative works

J2 Creative work included in group exhibition, performance, recording or anthology

J3 Exhibition curatorship

K Other academic outputs (in categories other than those listed above)

G4 Have any of the investigators or their works won prizes, awards or other major tributes as a result of the research project? Yes
If yes, give details.

G5 Research commercialisation

Provide details of commercialisation resulting from the research project against the categories listed below.

Commercialisation Types				
Patent				
patents establish legally enforceable protection of rights over intellectual property associated with inventions)				
Patents filed				
Type	Where lodged	Application number	Title	
Patents pending				

Type	Where lodged	Application number	Title
Invention disclosure			
an invention disclosure occurs when a device, substance, method or process that is apparently new, useful and involves an inventive step is made known to personnel within an institution who have responsibility for managing the institution's patenting and research commercialisation activities			
		Invention disclosures	
Plant breeder right			
a plant-breeder right is a temporary monopoly granted to a plant breeder seeking registration of a new plant variety. It gives the breeder the exclusive right to produce and reproduce the propagating material of the variety and to stock, sell, import or export the propagating material. In order for a plant-breeder right to be granted, the variety must be novel, distinct, uniform and stable.			
		Plant Breeder rights	
Application number for each filed	Certificate number for each granted		
Licence executed (include licences, options and assignments)			
A licence agreement formalises the transfer of technology between two parties, where the owner of the technology (the licensor) permits the other party (the licensee) to share the rights to use the technology. An option agreement grants the potential licensee a time period during which it may evaluate the technology and negotiate the terms of a licence agreement. An assignment agreement conveys all right, title and interest in and to the licensed subject matter to the named assignee.			
		Licences executed	
Start-up company formed			
<i>'start-up companies' refers to companies engaged in businesses that were dependent, for their formation, upon licensing or assignment of technology developed in this research project.</i>			
		Start-up companies formed	
Name of company	ABN		
Describe any other commercial outcomes from the project			

G6 Evidence of impact and contribution. Is there evidence that this project has had an impact in the research field or the broader public domain? Yes
If yes, briefly describe the impact and contribution.

The ARC has access to standard citation data on articles published in ISI journals. However, there may be other indicators of impact including, for example, citations to books, re-publication, translations, reviews, invited keynote addresses, other invitations, newspaper/media /expert commentary or advice to Government.

Three of the CI papers were published in "Avances in Mathematics" one of the most influential journals in Mathematics.

CI also published a paper in a new but highly regarded "Journal of Noncommutative Geometry" and submitted a paper (with Markl) to an influential journal "Geometry and Topology".

CI was invited to give a plenary talk at the 50th Anniversary Meeting of Australian Mathematical Society in September 2006.

CI and his research assistant Mark Weber were invited speakers on many International Conferences.

The impact of our research is also evident from the fact that in April 2008 there was a special conference in Barcelona devoted to higher operads, which is the new direction founded by CI and the main subject of study of the project. CI gave 3 one hour lectures and Mark Weber gave 2 one hour lectures on this conference.

Part H RESEARCH TRAINING, CAREERS AND EMPLOYMENT

H1 Postgraduate research training – Australian postgraduates supported by this project

Stipends: include students receiving ARC-funded stipends under Discovery and Linkage, but exclude DEST Australian Postgraduate Awards (APAs)

Research support: include students whose postgraduate or Honours research is assisted by ARC funding (for example, use of equipment funded under LIEF, international research experience funded under Linkage-International, and project support provided under Discovery-Projects or Linkage-Projects)

Number of PhD students receiving stipends and project support	
Number of research Masters students receiving stipends and project support	
Number of PhD students receiving project support but not stipends	
Number of Masters students receiving project support but not stipends	
Number of Honours students receiving project support but not stipends	

H2 Postgraduate research training – Overseas postgraduates supported by this project

Research support: include students enrolled in overseas universities whose postgraduate or Honours research is assisted by ARC funding (for example, use of equipment funded under LIEF, international research experience funded under Linkage-International)

Number of overseas PhD students involved in the project	
Number of overseas research Masters students involved in the project	
Number of overseas Honours students involved in the project	

H3 Early career researchers

Early career researchers are researchers with up to 5 years postdoctoral experience

ECRs named in the application
no ECRs named in the application

Did any other ECRs participate? If so, how many? Not selected

H4 Other employed personnel

Number of research associates/assistants funded (full time)	1
Number of research associates/assistants funded (part time)	
Number of professional and/or technical officers	
Industry Partner employees (not PIs)	
Other personnel involved (provide details below)	

Part I FUNDING CONTRIBUTIONS

Part I is not applicable to DP scheme.

Part J OTHER FUNDING

J1 Income

[Include only those income sources identified in the application form]

Source - DIRECT COSTS only	Request/commitment in application	Actual contribution
ARC Discovery funding	261046	156143
University/ies		
Other (provide details in section below relating to Actual 'Other' contribution only)		
	218898	34000
TOTAL - DIRECT COSTS only	479944	190143

J2 Has funding from other sources, including other ARC grants, supported this research? Not selected
If yes, provide details of the source and amount of funding received, under the following headings:

	Details of source/grant type	Funding received (\$AUS)
Host Institution		
	Total Host Institution support	0
ARC Funding Programs		
	Total other ARC Funding Programs	0
Other Government Competitive Funds		
	Total other Government competitive funds	0
Other Government Non-Competitive Funds		
	Total other Government Non-Competitive funds	0
Other funds from industry		
	Total other industry funds	0
Other external support		
	Max Plank-Istitut fur Mathematik research shcolarship (December 2007 - March 2008)	24000
	Centre de Recerca Matematica (Barcelona, Spain) research shcolarship (April 2008 - June 2008)	10000
	Max Plank-Istitut fur Mathematik research shcolarship (January 2009 - February 2010)	10000
	Total other external support	44000
	GRAND TOTAL	44000

Part K

Thank you for completing the final report.

The ARC may contact you if clarification or further information is required to acquit the grant.

Information on this form is collected in order to determine whether the research project funded by the ARC has reached satisfactory completion and for post award reporting.

Researchers should note that if the Final Report is not submitted on time, or if the ARC is not satisfied with the outcomes of the Project, this will be noted against any further Applications under any ARC program submitted by, or on behalf of, the Chief Investigator or Fellow and will be taken into account in the assessment of those applications. Applications under any ARC program will be deemed ineligible if submitted by, or on behalf of a Chief Investigator or Fellow on a Project for which the Final Report is outstanding.

Information in the Final Reports enables the ARC to account for the public funding of research and to promote the value of research to the Australian community. The ARC compiles statistical reports on the outcomes of publicly-funded research, drawing on the information you have provided in this Final Report.

Drawing on the information in the Final Report on the results, outcomes and significance of the research, the ARC may identify projects which have broad interest for media coverage and/or inclusion in ARC publications including the ARC website. The ARC will endeavour to contact you should it wish to publicise the results of your research, but bear in mind that this may not always be possible.